Brief Monad Example

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Outline

IO PROBLEM AS MOTIVATION

Suppose we have a relation R between integers and files. R simply takes an interger n and writes it to a file f.

Denote by $f[n]$ the file that results from writing the integer n to the file f. That is, $\langle n, f[n] \rangle \in R$.

Notice that $\langle n, f[n][n] \rangle$ is also in R. Thus there are two values for the integer n in R, and so R is not a function. That is, the naive approach to IO computation is not functional.

For a language to be purely functional, we need a more sophisticated approach to IO, and computation in general.

COMPUTATIONS AS FUNCTORS

The key insight into achieving pure functionality is treating values as objects of a category, and computations as functors between values.

For example, let A be an object of values (of type A). We can treat various computations on values in A as a functor M:

- **Exceptions**: $MA = (A + E)$ (E a set of exceptions).
- **Side effects**: $MA = (A \times S)^S$ (S a set of states).
- **Continuations**: $MA = R^{(R^A)}$ (R a set of results).

KLEISLI TRIPLES

We foramilze the insight as follows:

KLEISLI TRIPLE (1)

A *Kleisli triple* over a category **C** is a triple $(M, \eta, *)$, where

- $M : Ob(\mathbf{C}) \to Ob(\mathbf{C})$
- $\eta_A : A \to MA$, for all $A \in Ob(\mathbf{C})$,
- $f^* : MA \to MB$, for all $f : A \to MB$

Kleisli Triples

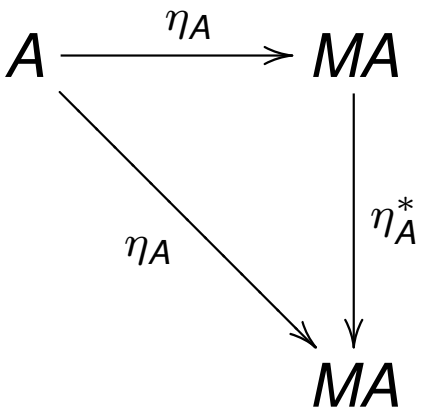
Kleisli Triple (2)

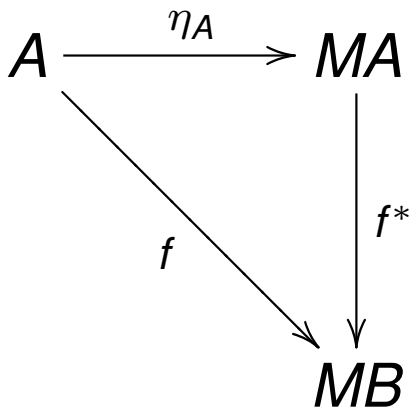
The following equations must be satisfied:

1. $\eta_A^* = \mathrm{id}_{MA}$
2. $f^* \circ \eta_A = f$, for all $f : A \to MB$
3. $g^* \circ f^* = (g^* \circ f)^*$, for all $f : A \to MB$ and $g : B \to MC$.

KLEISLI TRIPLES

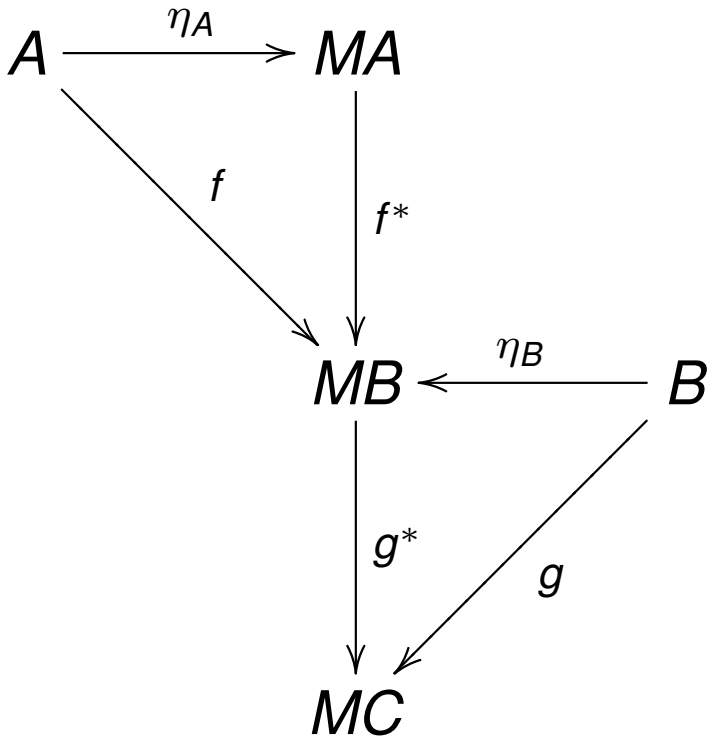
Equations 1. and 2. are visualized as





KLEISLI TRIPLES

Equation 3. is visualized as



KLEISLI TRIPLES

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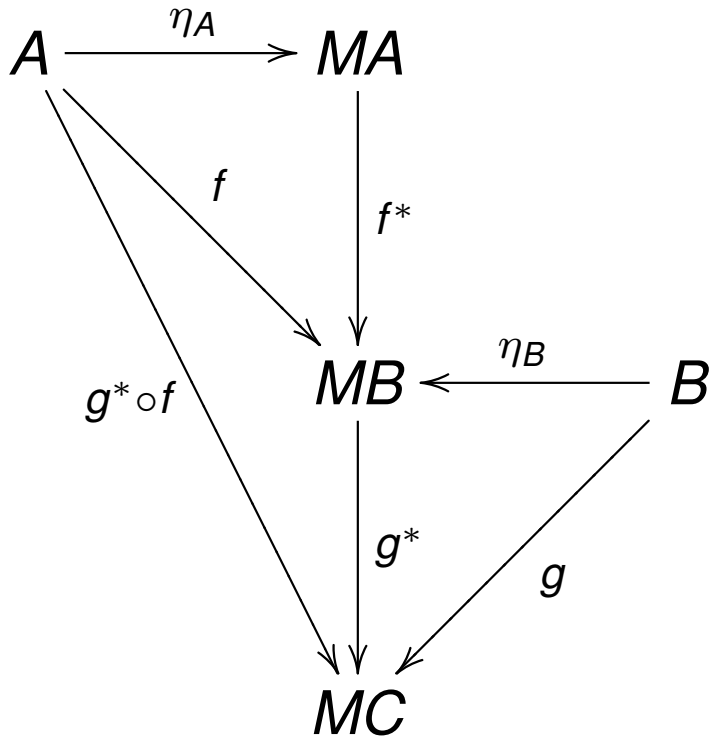

KLEISLI TRIPLES

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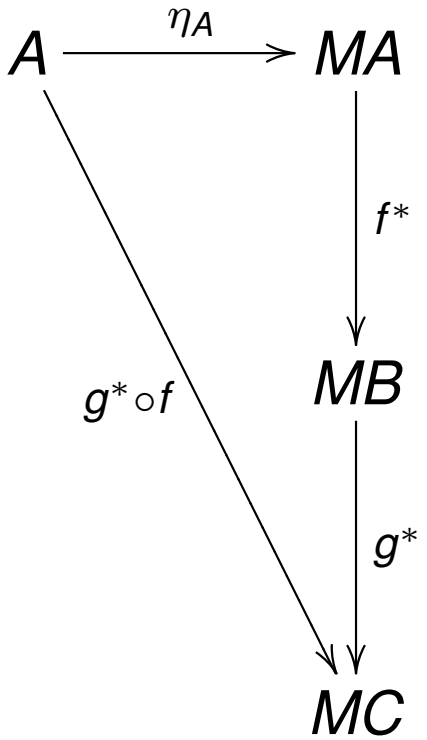


HASKELL EXAMPLE

Haskell monads are Kleisli triples. They are called monads since every Kleisli triple can be extended to a category theoretic monad (a kind of endofunctor we won't define).

EXCEPTION HANDLING

An Exception monad *(Exc, eta, ast)* can be defined as follows:

- Exc a = ThrowException | Iden a
- eta :: a -> Exc a
  eta a = Iden a
- ast f = f, for all *f* :: a -> Exc b

HASKELL EXAMPLE

EXCEPTION HANDLING

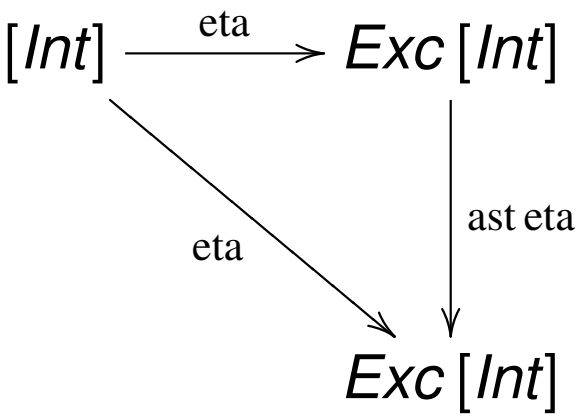
Let fstItem be a function that returns the first item in a nonempty list. This function is not defined on the empty list. Thus fstItem ([ ]) should throw an exception.

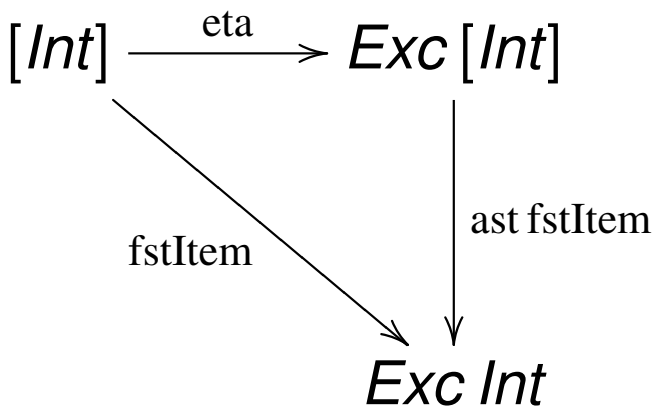
We can use the Exception monad to achieve the desired behavior:

- fstItem :: [a] -> Exc a
- fstItem ([ ]) = ThrowException
- fstItem (a:as) = Iden a

HASKELL EXAMPLE

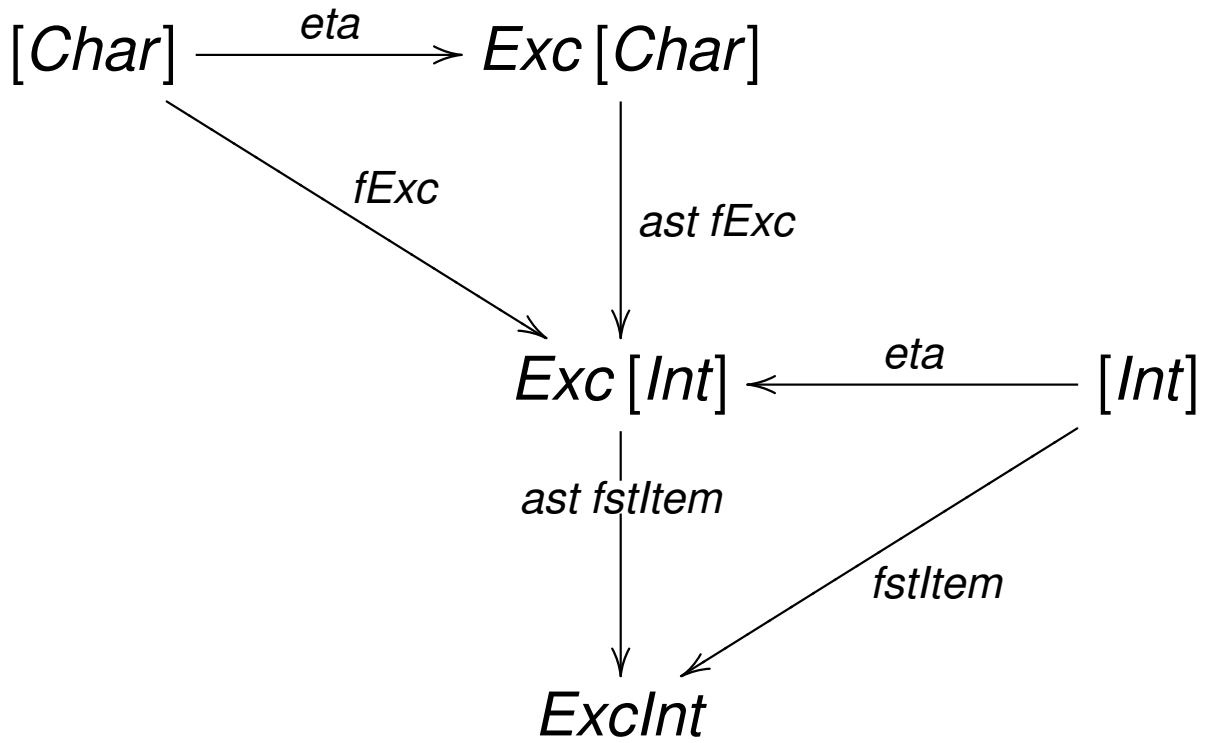
We need to verify that eta and ast satisfy Kleisli Equations 1. 2. and 3. We do so for fstItem on [Int]. Equations 1. and 2. are as follows:





HASKELL EXAMPLE

To verify Equation 3, consider fExc :: [Char] -> Exc [Int], which maps lists of chars to lists of ints, throwing an exception on the empty list. We then have:



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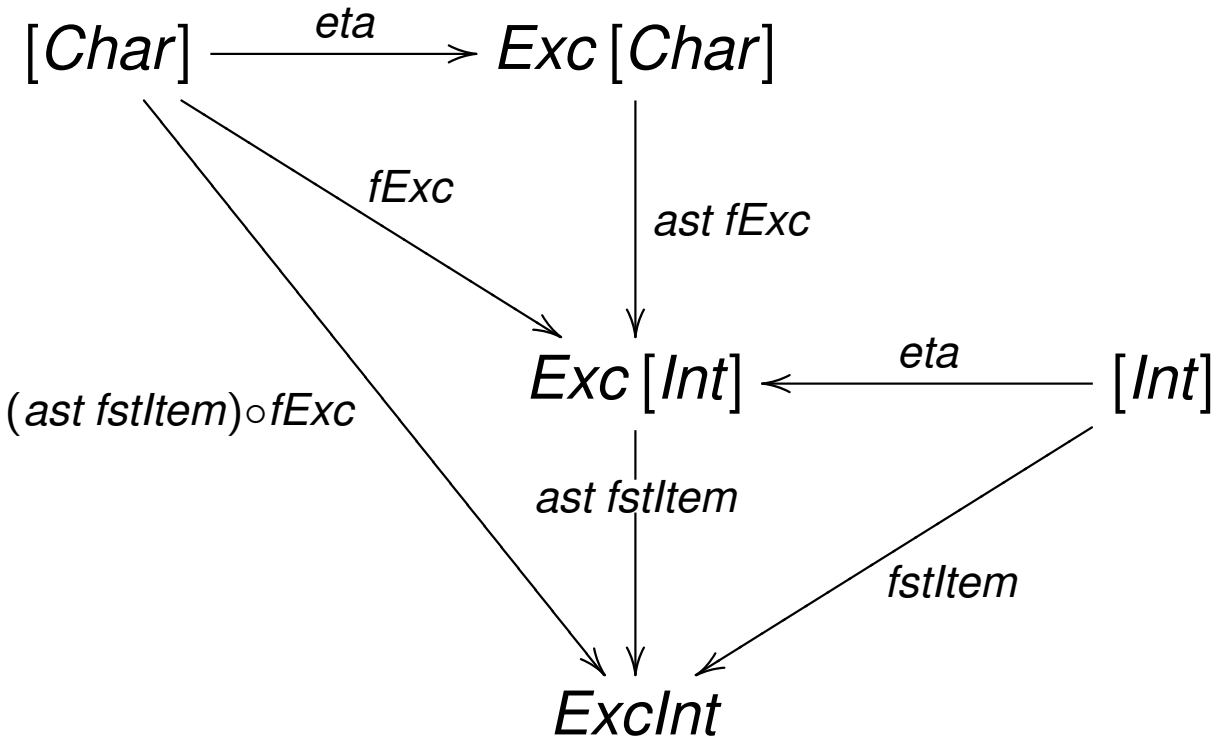

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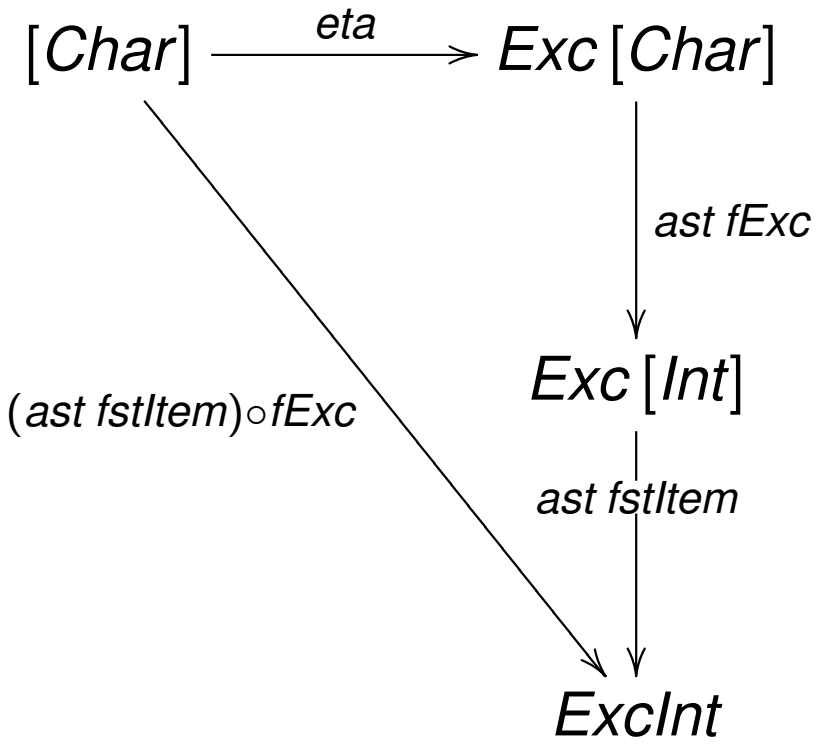

HASKELL SYNTAX

BIND AND RETURN

In haskell jargon:

- $*$ (ast) is called 'bind', and is represented by »=
- η (eta) is called 'return', and is represented by return

These are the building blocks of monadic computation. To become a super haskell guru, you must master them.

For Great Good

The Haskell

More information on Haskell Syntax and Theory:

- The Haskell: http://www.haskell.org
- Haskell Beginners: http://www.haskell.org/mailman/listinfo/beginners
- Haskell Cafe: http://www.haskell.org/mailman/listinfo/haskell-cafe