1. [10 pts.] Write an equation for a full joint distribution in terms of the following models:
   - $\theta_{SKF}$, for a student knowing the fact that answers a question,
   - $\theta_{SRH}$, for the student raising his or her hand,
   - $\theta_{SCA}$, for the student correctly answering the question,

   with conditional dependencies as shown in the following network:

   ![Diagram](image)

2. [10 pts.] Draw or describe a graphical representation of an extension of the above probability model, using random variables for:
   - student listening to lesson explaining fact
   - teacher asking question
   - student hearing question

   Justify each additional conditional dependency in a sentence (for example: ‘a student is more likely to raise his or her hand if he or she knows the answer’).

3. [10 pts.] PROGRAMMING: Write a program to read in models of language change over generations of speakers. Use the following format for component models of a grandparent speaker $G$, a parent speaker $P$ (given grandparent), and a child speaker $C$ (given parent) making use of the word ‘who’ as opposed to ‘whom’ in the position of an accusative filler (e.g. ‘who/whom did you invite?’):

   ```
   G : who = .1
   G : whom = .9
   
   P who : who = 1
   P who : whom = 0
   P whom : who = .2
   P whom : whom = .8
   ```
then use these models to calculate a conditional probability distribution table for $P(P|C)$, and print it in the following format:

\[ P\text{giv}C \text{ who : who } = 0.4375 \]
\[ P\text{giv}C \text{ who : whom } = 0.5625 \]

4. [10 pts.] PROGRAMMING: Write a program to read in models for all variables $R$, $W$, and $O$ in the ‘repeated trials’ model shown at the beginning of the lecture notes on sequence modeling, in the following format

- $R: \text{ohio} = .5$
- $R: \text{phil} = .5$
- $W: /\text{nek}/ = .6$
- $W: /\text{naek}/ = .4$
- $O \text{ohio} /\text{nek}/ : [\text{nek}] = 1$
- $O \text{phil} /\text{nek}/ : [\text{nek}] = .667$
- $O \text{phil} /\text{nek}/ : [\text{naek}] = .333$
- $O \text{ohio} /\text{naek}/ : [\text{naek}] = 1$
- $O \text{phil} /\text{naek}/ : [\text{naek}] = 1$

and an input sequence of any number of observations in the format:

\[ I [\text{naek}] [\text{nek}] [\text{naek}] ... \]

then print out a probability distribution for $R$ given all of these input observations, in the following format (note: probabilities given observations should not necessarily match initial $R$ model):

\[ R\text{giv}I\text{data} : \text{ohio} = .4 \]
\[ R\text{giv}I\text{data} : \text{phil} = .6 \]

Your program should be as short as possible. Hand in all inputs and outputs.