LING5702: Problem Set 1

Due via Carmen dropbox at 11:59 PM 1/23.

1. Assume a probability space, as described in the lecture notes, over a seed you are planting, with outcomes:

   - T that the plant turns out to be a tree
   - S that the plant turns out to be a shrub
   - G that the plant turns out to be green
   - B that the plant turns out to be brown

   Using the above outcomes ...

   (a) [3 pts.] Write a probability expression expressing that half the time your seed will turn out to be green.

   (b) [3 pts.] Write a probability expression expressing that half the time your seed will turn out to be a green tree.

   (c) [3 pts.] Write a probability expression expressing that if your seed turns out to be green, half the time it will be a tree.

2. Suppose you have the following probability model over a seed you are planting:

   - a sixth of the time you get a green tree
   - a third of the time you get a brown tree
   - a sixth of the time you get a green shrub
   - a third of the time you get a brown shrub

   Using the above model ...

   (a) [3 pts.] What is the probability it will turn out to be a green plant (either a tree or a shrub)?

   (b) [3 pts.] If you already know it will turn out to be green, what is the probability it will turn out to be a tree?

3. Using the generalized quantifier functions in the lecture notes on typed lambda calculus and the following predicates:

   - Tree x, meaning that x is a tree
   - Green x, meaning that x is green
   - Round x, meaning that x is round
   - Park x, meaning that x is a park area
   - In x y, meaning that x is in area y

   (a) [3 pts.] Write a typed lambda calculus expression stating that half the round trees are green.

   (b) [3 pts.] Write a typed lambda calculus expression stating that half the trees are green and round.

   (c) [3 pts. – difficult!] Write a typed lambda calculus expression stating that all the trees are in some park (possibly a different park for each tree).

4. [difficult!] Using the non-intensional and intensional quantifier functions from the lecture notes on typed lambda calculus:

   - $\text{Ratio}_{\geq}$ n R S, meaning that at least n (fraction) of R are in S,
   - $\text{Count}_{\geq}$ n R S, meaning that at least n (instances) of R are in S,
   - $\text{IntensionOfCount}_{\geq}$ i n R S, meaning that i is an intension that at least n of R are in S,

   and the following predicates:

   - Kid x, meaning that x is a kid,
   - Horse y, meaning that y is a horse,
   - Time t, meaning that t is a point in time,
   - Ride t x y, meaning that x rides y at time t,
   - Want t x i, meaning that x wants intension i to be true at time t,

   write a typed lambda calculus expression stating that:

   (a) [2pts.] Every kid wants to ride a horse at some point in time (but they don't care which horse or when);

(b) [2pts.] For every kid there is a particular real horse that they want to ride at some point in time (but they don't care when, and it may be a different horse for each kid).

(c) [2pts.] There is a single (presumably famous) horse that every kid wants to ride at some point in time;

Note: only quantifier functions can take lambda functions $(\lambda_x \dots)$ as arguments; all the predicates can take only entity variables as arguments.