# Ling 5701: Lecture Notes 6
# A Model of Complex Ideas in Associative Memory

We have seen how interconnected neurons can define mental states and cued associations.

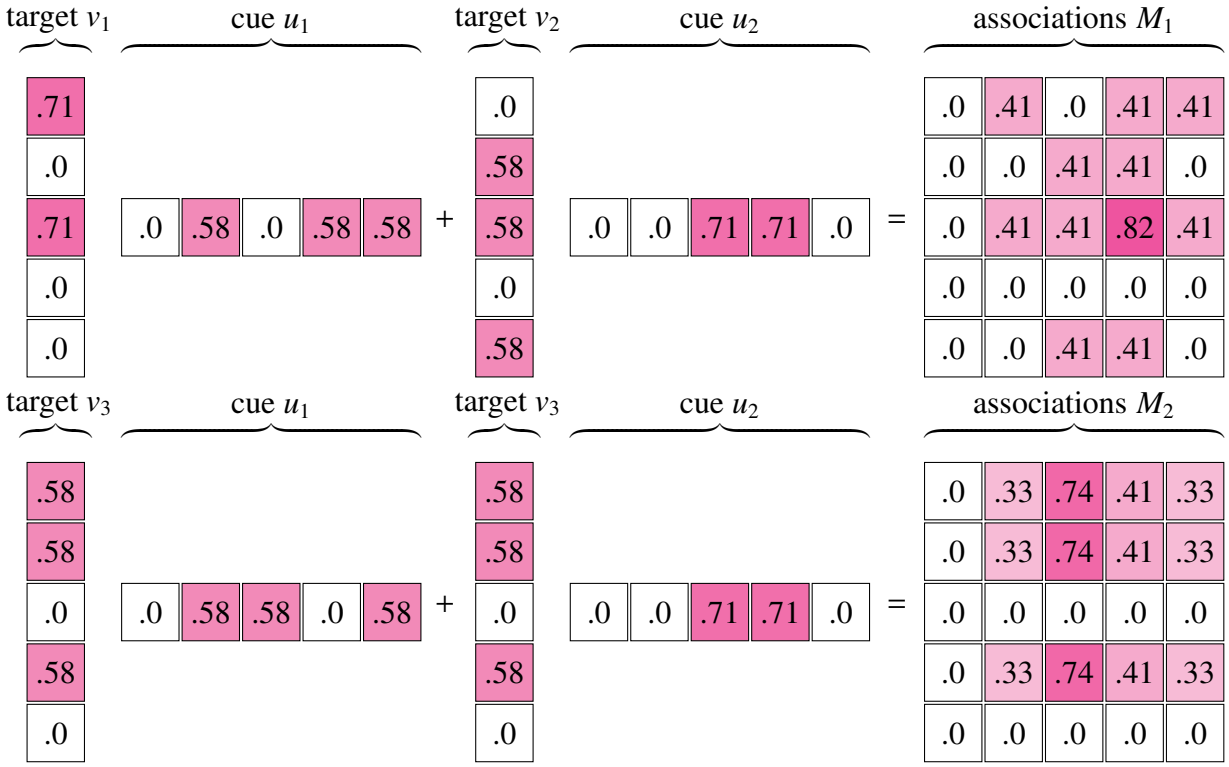
This lecture will describe how mental states and cued associations can define complex ideas.

## Contents

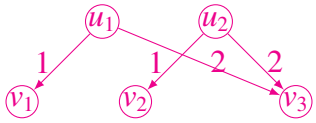


### 6.1 Previously: mental states and cued associations

Recall neural activation patterns and potentiated connection weights:



define coordinates of points (mental states) in mental space, linked by cued associations:




## 6.2 Referential states and elementary predications

Cued associations among mental states can be used to build complex ideas:

- Mental states are inherently **referential** [Karttunen, 1976]: e.g. think of the food you ate today.

  These **generalize** over objects/stimuli that produce the same pattern of activation (appearance in occipital lobe, smell in olfactory cortex, etc.).

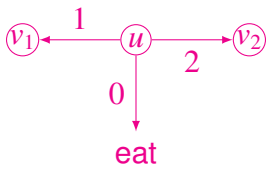
- Some states form **elementary predications** [Copestake et al., 2005], e.g. chimps eating nuts.

  These generalize over event stimuli (movements in motor cortex, sound effects in auditory cortex) and relationships between **participants** (eater, food).

  These must have a specific **type** (like eat or hit, notated as a '0' association), which may be characteristic features of the vector, in order to define participants' roles.

- Cued associations then connect elementary predications with participating referential states (notated with cued associations labeled '1', '2', etc.).

If we draw cued associations as labeled directed edges, elementary predications look like this:




where $v_1$ is a generalization over eaters and $v_2$ is a generalization over food, and $u$ is a generalization over elementary predications (events) of type 'eat.'

This can also be written $(M_0\ u) = \mathsf{eat}, (M_1\ u) = v_1, (M_2\ u) = v_2$ or simply $(\mathsf{eat}\ u\ v_1\ v_2)$.

Elementary predications are referential states, can be described by other elementary predications...

- temporal predications: date, time, duration, speed $(\mathsf{during}\ u\ u'\ u''), (\mathsf{hit}\ u''\ v_2\ v_3)$
- implicational predications: one elementary predication causes another $(\mathsf{cause}\ u\ u'\ u''), (\mathsf{break}\ u''\ v_2)$
- predications about belief (theory of mind): $(\mathsf{believe}\ u\ v_1\ u'), (\mathsf{Koko}\ u'\ v_1)$

Referential states and elementary predications are pre-linguistic, and differ from nouns and verbs. (Also, elementary predications can be encoded as nouns: *Rome's destruction of Carthage*.)

## 6.3 Broad and narrow generalizations in referential states

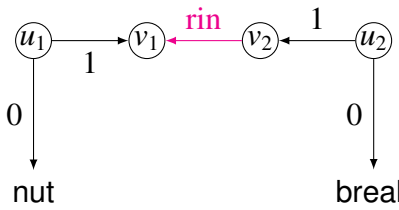
Complex ideas often define narrow generalizations by combining multiple broad generalizations.

- E.g.: *nuts break* (defines a subset of broken nuts from nuts and broken things)

The narrow (broken nuts) and one broad (nuts) generalization seem to be distinct, but not the other:

- *nuts break, but they are delicious* (subsequent reference to broken nuts)
- *nuts break, but usually they don't break* (subsequent reference to nuts)
- ?? *nuts break, but usually they aren't nuts* (subsequent reference to broken things)

so we assume just one broad ($v_1$) and one narrow ($v_2$) generalization persist as referential states:



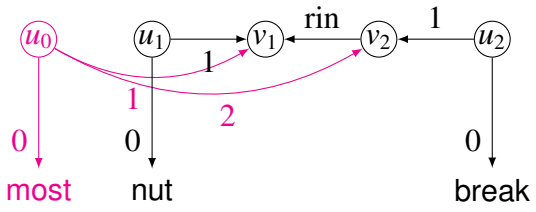

Here $v_1$,$v_2$ generalize over nuts and broken nuts ($u_1$,$u_2$ generalize over being a nut and breaking).

The narrow generalization restricts and inherits from the broad one ('restriction inheritance': rin).

## 6.4 Quantifiers

Sometimes narrow and broad generalizations are compared by cardinality.

- E.g.: *'most nuts break'* asserts that the set of broken nuts will include over half of all nuts.




These are elementary predications called **quantifiers** [Barwise and Cooper, 1981].

(The broader generalization is the **restrictor**, and the narrower is the **nuclear scope**.)

Quantifiers are important because they define **conditional probabilities**, essential for planning.

- E.g.: Most kola nuts break, but few panda nuts break. (Or: . . . usually / rarely).

Quantifiers may be inferred from the strength (frequency) of cued associations between referents.

Quantifier *words* are learned late: 'quantifier spreading' [Inhelder and Piaget, 1958, Philip, 1995].

Children until about 10 years old don't reliably constrain the restrictor with the noun, etc.:

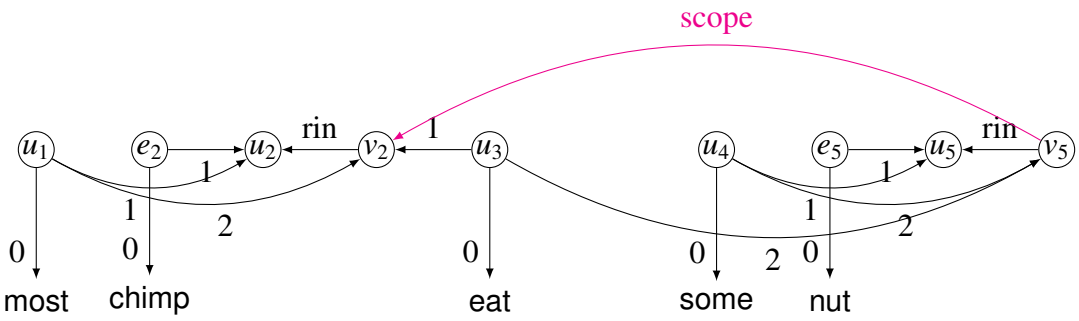
- *Does every bike have a basket?* (shown picture with three bikes, four baskets) → answer: *No.*

But this may indicate a lack of mapping from words to ideas, rather than a lack of this kind of idea.

## 6.5 Quantifier scope

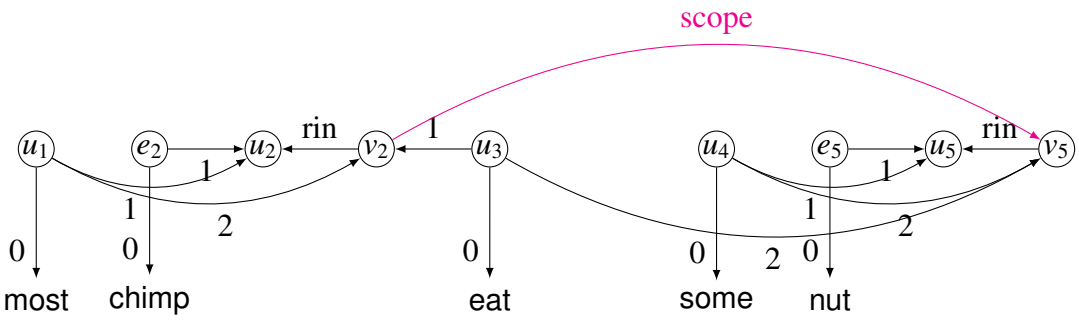
If multiple quantified referents are connected by predications, they must be scoped.

This distinguishes whether there is a set of nuts for each chimp or a set of chimps for each nut:




In the above example, there is a generalization over eaten nuts for each eater chimp.

In the below example, there is a generalization over eater chimps for each eaten nut (i.e. there is some nut, say a coconut, that everyone shared):




## 6.6 How are complex ideas experienced?

Remember, in this model a complex idea is a collection of cued associations in associative memory.

The entire idea is not all active at the same time.

How is such an idea experienced?

Just as we apprehend visual scenes by saccading from one physical fixation point to another, in this model we apprehend complex ideas by **transitioning** from one referential state to another, via cued associations.

So, as we think about chimps and nuts, there is always a 'you-are-here pointer' in the graph.

## 6.7 Cued association graphs translate into lambda calculus

Philosophers often define complex ideas using logical expressions in lambda calculus.

Lambda calculus is beyond the scope of this course, but if you happen to already know it...

The cued association graphs above can be translated into lambda calculus (not a cognitive process).

1. Add a lambda term to $\Gamma$ for each predication in $\Gamma$ with no outscoped variables or inheritances:

$$\frac{\Gamma, (f\ v_0\ v_1\ ..\ v\ ..\ v_N)\ ;\ \Delta}{\Gamma, (\lambda_v\ f\ v_0\ v_1\ ..\ v\ ..\ v_N)\ ;\ \Delta}\ f \notin Q,\ \forall_u\ (\mathbf{M}_{\text{scope}}\ u){=}v \notin \Gamma,\ \forall_{f \in \{\mathbf{M}_{\text{rin}}, \mathbf{M}_{\text{cin}}, \mathbf{M}_{\text{uin}}\}}\ (f\ v){=}u \notin \Gamma \quad \text{(P)}$$

2. Conjoin lambda terms over the same variable in $\Gamma$ (this combines modifier predications):

$$\frac{\Gamma, (\lambda_v\ \phi), (\lambda_v\ \psi)\ ;\ \Delta}{\Gamma, (\lambda_v\ \phi \wedge \psi)\ ;\ \Delta} \quad \text{(C)}$$

3. Move terms in $\Gamma$ with no missing predications, outscoped variables or inheritances to $\Delta$:

$$\frac{\Gamma, (\lambda_v\ \psi)\ ;\ \Delta}{\Gamma\ ;\ (\lambda_v\ \psi), \Delta}\ \forall_{f' \notin Q}\ (f'..\ v\ ..) \notin \Gamma,\ \forall_u\ (\mathbf{M}_{\text{scope}}\ u){=}v \notin \Gamma,\ \forall_{f \in \{\mathbf{M}_{\text{rin}}, \mathbf{M}_{\text{cin}}, \mathbf{M}_{\text{uin}}\}}\ (f\ v){=}u \notin \Gamma \quad \text{(M)}$$

4. Add translations $\tau_f$ of quantifiers $f$ in $\Gamma$ over complete lambda terms in $\Delta$:

$$\frac{\Gamma, (f\ p\ u\ v)\ ;(\lambda_u\ \phi), (\lambda_v\ \psi), \Delta}{\Gamma, (\tau_f\ (\lambda_u\ \phi)\ (\lambda_v\ \psi))\ ;(\lambda_u\ \phi), (\lambda_v\ \psi), \Delta}\ f \in Q,\ \forall_{f' \in \{\mathbf{M}_{\text{rin}}, \mathbf{M}_{\text{cin}}, \mathbf{M}_{\text{uin}}\}}\ (f'..){=}v \notin \Gamma,\ \forall_{f' \in Q}\ (f'..\ v\ ..) \notin \Gamma \quad \text{(Q1)}$$

$$\frac{\Gamma, (\mathbf{M}_{\text{scope}}\ v){=}v', (\tau_f\ (\lambda_u\ \phi)\ (\lambda_v\ \psi))\ ;\ \Delta}{\Gamma, (\lambda_{v'}\ \tau_f\ (\lambda_u\ \phi)\ (\lambda_v\ \psi))\ ;\ \Delta}\ \forall_{u'}\ (\mathbf{M}_{\text{scope}}\ u){=}u' \notin \Gamma \quad \text{(Q2)}$$

$$\frac{\Gamma, (\mathbf{M}_{\text{scope}}\ u){=}u', (\mathbf{M}_{\text{scope}}\ v){=}v', (\tau_f\ (\lambda_u\ \phi)\ (\lambda_v\ \psi))\ ;\ \Delta}{\Gamma, (\mathbf{M}_{\text{scope}}\ u){=}u', (\lambda_{v'}\ \tau_f\ (\lambda_u\ (\lambda_{u'}\ \phi)\ v')\ (\lambda_v\ \psi))\ ;\ \Delta} \quad \text{(Q3)}$$

5. Add a lambda term to $\Gamma$ for each inheritance that is empty or from complete term in $\Delta$:

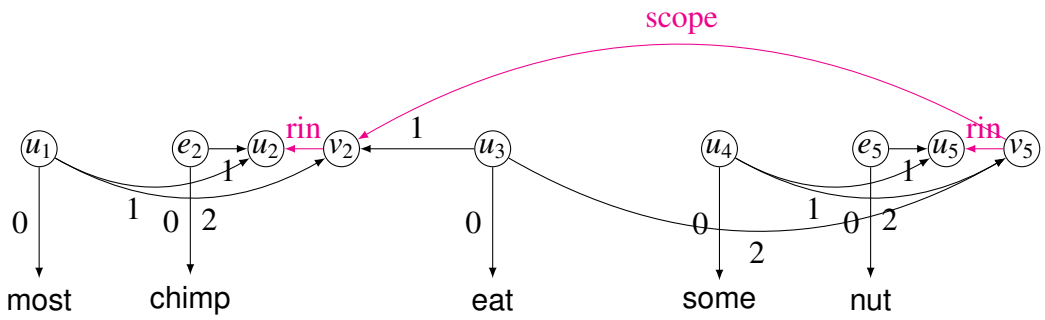
$$\frac{\Gamma, (f\ v){=}u\ ;\ \Delta}{\Gamma, (f\ v){=}u, (\lambda_u\ \mathsf{True})\ ;\ \Delta}\ f \in \{\mathbf{M}_{\text{rin}}, \mathbf{M}_{\text{cin}}\},\ \forall_{f' \notin Q}\ (f' ..\ u\ ..) \notin \Gamma \qquad \text{(I1)}$$

$$\frac{\Gamma, (f\ v){=}u\ ;\ (\lambda_u\ \phi), \Delta}{\Gamma, (\lambda_v\ (\lambda_u\ \phi)\ v)\ ;\ (\lambda_u\ \phi), \Delta}\ f \in \{\mathbf{M}_{\text{rin}}, \mathbf{M}_{\text{cin}}\},\ \forall_{u'}\ (\mathbf{M}_{\text{scope}}\ u){=}u' \notin \Gamma \qquad \text{(I2)}$$

$$\frac{\Gamma, (\mathbf{M}_{\text{scope}}\ u){=}u', (f\ v){=}u\ ;\ (\lambda_u\ \phi), \Delta}{\Gamma, (\mathbf{M}_{\text{scope}}\ u){=}u', (\mathbf{M}_{\text{scope}}\ v){=}u', (\lambda_v\ (\lambda_u\ \phi)\ v)\ ;\ (\lambda_u\ \phi), \Delta}\ f \in \{\mathbf{M}_{\text{rin}}, \mathbf{M}_{\text{cin}}\},\ \forall_{v'}\ (\mathbf{M}_{\text{scope}}\ v){=}v' \notin \Gamma \qquad \text{(I3)}$$

$$\frac{\Gamma, (\mathbf{M}_{\text{scope}}\ u){=}u', (\mathbf{M}_{\text{scope}}\ v){=}v', (f\ v){=}u\ ;\ (\lambda_u\ \phi), \Delta}{\Gamma, (\mathbf{M}_{\text{scope}}\ u){=}u', (\mathbf{M}_{\text{scope}}\ v){=}v', (\lambda_v\ (\lambda_u\ (\lambda_{u'}\ \phi)\ v')\ v)\ ;\ (\lambda_u\ \phi), \Delta}\ f \in \{\mathbf{M}_{\text{rin}}, \mathbf{M}_{\text{cin}}\} \qquad \text{(I4)}$$

For example, our graph:




translates into:

$(\text{most}\ p_1\ u_2\ v_2), (\mathbf{M}_{\text{scope}}\ v_5){=}v_2, (\text{some}\ p_4\ u_5\ v_5), (\text{chimp}\ e_2\ u_2), (\text{nut}\ e_5\ u_5), (\text{eat}\ u_3\ v_2\ v_5)\ ;$

$P\ (\text{most}\ p_1\ u_2\ v_2), (\mathbf{M}_{\text{scope}}\ v_5){=}v_2, (\text{some}\ p_4\ u_5\ v_5), (\lambda_{u_2}\ \text{chimp}\ e_2\ u_2), (\lambda_{u_5}\ \text{nut}\ e_5\ u_5), (\lambda_{v_5}\ \text{eat}\ u_3\ v_2\ v_5)\ ;$

$M\ (\text{most}\ p_1\ u_2\ v_2), (\mathbf{M}_{\text{scope}}\ v_5){=}v_2, (\text{some}\ p_4\ u_5\ v_5)\ ;\ (\lambda_{u_2}\ \text{chimp}\ e_2\ u_2), (\lambda_{u_5}\ \text{nut}\ e_5\ u_5), (\lambda_{v_5}\ \text{eat}\ u_3\ v_2\ v_5)$

$Q1\ (\text{most}\ p_1\ u_2\ v_2), (\mathbf{M}_{\text{scope}}\ v_5){=}v_2, (\text{some}\ (\lambda_{u_5}\ \text{nut}\ e_5\ u_5)\ (\lambda_{v_5}\ \text{eat}\ u_3\ v_2\ v_5))\ ;\ (\lambda_{u_2}\ \text{chimp}\ e_2\ u_2), (\lambda_{u_5}\ ...), (\lambda_{v_5}\ ...)$

$Q2\ (\text{most}\ p_1\ u_2\ v_2), (\lambda_{v_2}\ \text{some}\ (\lambda_{u_5}\ \text{nut}\ e_5\ u_5)\ (\lambda_{v_5}\ \text{eat}\ u_3\ v_2\ v_5))\ ;\ (\lambda_{u_2}\ \text{chimp}\ e_2\ u_2), (\lambda_{u_5}\ ...), (\lambda_{v_5}\ ...)$

$M\ (\text{most}\ p_1\ u_2\ v_2)\ ;\ (\lambda_{v_2}\ \text{some}\ (\lambda_{u_5}\ \text{nut}\ e_5\ u_5)\ (\lambda_{v_5}\ \text{eat}\ u_3\ v_2\ v_5)), (\lambda_{u_2}\ \text{chimp}\ e_2\ u_2), (\lambda_{u_5}\ ...), (\lambda_{v_5}\ ...)$

$Q1\ (\text{most}\ (\lambda_{u_2}\ \text{chimp}\ e_2\ u_2)\ (\lambda_{v_2}\ \text{some}\ (\lambda_{u_5}\ \text{nut}\ e_5\ u_5)\ (\lambda_{v_5}\ \text{eat}\ u_3\ v_2\ v_5)))\ ;\ (\lambda_{u_2}\ ...), (\lambda_{u_5}\ ...), (\lambda_{v_5}\ ...)$

# References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