HYPERINTENSIONAL QUESTIONS

Carl Pollard
INRIA-Lorraine and Ohio State University

WoLLIC 2008
Edinburgh, July 3, 2008

The handout for this talk is available at:

`http://www.ling.ohio-state.edu/~pollard/cvg/hyperq.pdf`

(1) **Basic Orientation of this Paper**

- **Possible-worlds semantics** (PWS) is well-established as a theory of natural-language (NL) meaning.
- But the standard version of PWS, which we will call **intensional** semantics, suffers from **foundational problems** whose gravity ranges from the merely annoying to the downright catastrophic.
- We review a proposed **alternative** to intensional semantics, called **hyperintensional** semantics, that solves many of these problems.
- Then we focus on consequences of "going hyper" for **questions** (meanings of interrogative sentences).

(2) **The relationship between Intensional and Hyperintensional Semantics**

- Intensional and hyperintensional semantics share a great many assumptions, as we will make clear.
- For example, unlike most other proposed alternatives to intensional semantics, hyperintensional semantics countenances **worlds** and **intensions** (functions from worlds).
- But unlike intensional semantics, in hyperintensional semantics, meanings of NL expressions are **not** intensions.
- Rather, they are **hyperintensions**, which are **more fine-grained** than intensions.
- But the intensions are still there: they are the **Stone duals** of the hyperintensions.

(3) **Shared Assumptions of Intensional and Hyperintensional Semantics**

- Basic **ontological** assumptions
- specific assumptions about **types of senses**
- specific assumptions about **types of extensions**

(4) **Shared Assumptions about Basic Ontology**

- There are different ways things might be, called **(possible) worlds**.
- One of the worlds, the **actual** world, is how things are.
- The philosophical stance called **hard actualism**, which holds that the actual world is the only world, is wrong.
- There are things called **senses**, that are independent of worlds, and these include the **meanings of NL expressions**.
- Every ordered pair of a sense and a world uniquely determines a thing called the **extension** of that sense at that world.
- The extension of an expression's meaning (at a world) is its **reference** (at that world).
- A sense is called **rigid** if it has the same extension at every world.

(5) **Shared Assumptions about Types of Senses**

- There are different **types** of senses.
- One type of sense is **propositions**.
- The meanings of **declarative sentences** are propositions.
- Another type of sense is **individual concepts**, or simply **individuals** for short).
- The meanings of **names** are **rigid** individuals.

(6) **Shared Assumptions about Types of Extensions**

- There are different types of extensions.
- One type of extension is **truth values**.
  - The **extensions of propositions** are truth values.
  - Therefore so are the **references of declarative sentences**.
  - There are exactly two truth values, true and false.
- Another type of extension is **entities**.
  - The **extensions of individuals** are entities.
  - Therefore so are the **references of names**.
  - Rigidity (the last assumption in (5)) means that **name refers to the same entity at every world**.

(7) **More Common Ground: Entailment and Equivalence**

- One proposition is said to **entail** another iff, at every world where its extension is true, the extension of the other is also true.
- For two NL sentences S1 and S2, whether sentence S2 **follows from** S1, which is an **empirical** question, is assumed to hinge on whether the proposition expressed by S1 entails the proposition expressed by S2.
- This makes entailment accessible to empirical investigation.
- Semantics is often defined as the science devoted to that investigation.
- Obviously entailment is a **preorder** (i.e. reflexive and transitive).
- Two propositions are said to be **(truth-conditionally) equivalent** iff they entail each other.

(8) **Is Truth-Conditional Equivalence Just Equality?**

- With a preorder, one can always ask: is it antisymmetric?
- To ask this about entailment is to ask whether truth-conditional equivalence is just equality of propositions.
- In terms of NL, this becomes the question of whether two sentences that follow from each other mean the same thing.
- Naive intuition screams **no**.
- Intensional semanticists (Lewis, Stalnaker, Montague, and the linguistic 'main stream') insist **yes**.
- Though here there have been plenty of dissenting voices: Thomason, Barwise and Perry, Chierchia and Turner, Muskens, Fox and Lappin to name just a few.
- How can we settle this?

(9) **A Fair Proposal**

- Notice that intensional semanticists **have no choice but to say yes** (that entailment is antisymmetric).
- That's because, for them, **entailment is the subset relation on sets of worlds**.
- But if your technical assumptions didn't **force** you to say **yes**, are there any facts about which sentences follow from which other sentences that would make you **want** to posit antisymmetry of entailment as an additional axiom?
- Frankly, I doubt it, but I'm willing to try to be open-minded.
- Hyperintensional semantics simply **makes no commitment** about whether entailment is antisymmetric.
- If you **really, really** had good reason to insist on it, nothing would stop you from adding an axiom that made it so.
- That's fair, isn't it?

(10) **Shared Technical Assumptions**

Technically, intensional and hyperintensional semantics have a lot in common:

- The semantic theory is asserted by nonlogical axioms (**meaning postulates**) in a **classical**, **two-valued**, **higher-order logic** (HOL) similar to the Church/Henkin simple theory of types.
- the basic type Bool (traditionally, t), for truth values, already supplied by the underlying logic
- the basic type Ent (traditionally, e), for entities
- acceptance of Henkin's axiom (explicitly rejected by Church) of **Boolean Extensionality** (here x and y are of type Bool):

  $\vdash \forall_{x,y}[(x \leftrightarrow\ y) \rightarrow\ (x = y)]$

  which identifies bi-implication with boolean equality.

(11) **Where Intensional and Hyperintensional Sematics Differ**
It is agreed that these three additional types are needed;

- World (traditionally, s)
- Prop for propositions (including sentence meanings)
- Ind, for individuals (including meanings of names)

but not what **status** these types should have.

- In **intensional** semantics, inspired by Kripke (1963), World is **basic**, while Ind and Prop are **defined**
- Whereas in **hyperintensional** semantics, inspired by Stone (1936, 1937), Jónsson and Tarski (1951), and Kripke (1959), World is **defined**, while Ind and Prop are **basic**.
- In this paper, we focus on World and Prop (see Pollard 2008a on the status of Ind).

(12) World **and** Prop **in Intensional Semantics**

- World is a **basic** type.
- Meanings are treated as **intensions** (functions from worlds).
- In particular, Prop is **defined** as World $\rightarrow$ Bool.
- So "p is true at w " is captured by $w \in p$.
- The boolean structure on propositions (the entailment order, the meanings of the "logic words") is **derived** from the classicality of the logic in which the theory is written!
- In particular, **entailment** is subset inclusion of sets of worlds.
- And so the antisymmetry of entailment is **forced**, no matter whether it is empirically motivated by facts about validity of NL arguments.
- Might it be, then, that defending the antisymmetry of entailment is just making a virtue of necessity?

(13) Prop **in Hyperintensional Semantics**

- Meanings are **not** intensions.
- Prop is a **basic** type.
- The empirically observed booleanness of NL reasoning is **asserted by nonlogical axioms of the theory of meanings**, not derived from the classicality of the logic in which the theory is written.
- We are free to axiomatize entailment as (merely) a boolean **preorder**.
- But people who **really** think entailment is antisymmetric can add an axiom to that effect and still enjoy the **other** virtues of hyperintensional semantics.

(14) World **in Hyperintensional Semantics**

- World is a **defined subtype** of Prop $\rightarrow$ Bool , and so "p is true at w" is captured by $p \in w$, **not** by $w \in p$.
- More specifically: worlds are sets of propositions which are **ultrafilters** with respect to the entailment preorder.
- We are able to say this because:
  - the HOL we use has Separation-style subtyping, along the lines of Lambek and Scott (1986); and
  - the property of being an ultrafilter is internally definable, so can be used to 'separate out' the ultrafilters from the other the sets of propositions.
  - But suptyping will turn out to have other applications too (such as defining types for questions).

(15) **Entailment in Hyperintensional Semantics**

- Pretheoretically, to say that p **entails** q is to say that no matter how things are, if p is true, then so is q.
- But now, worlds are just ultrafilters of propositions , so this amounts to $p \leq q$ (where $\leq$ is the boolean preorder).
- That's because of **Stone's Lemma** (a theorem of ZFC):

  In a boolean preorder, if every ultrafilter with p as a member also has q as a member, than $p \leq q$.
- To paraphrase: "boolean preorders act just like the pretheoretical notion of entailment, as long as you identify the notion of 'how things are' with 'ultrafilter'.
- This makes perfect intuitive sense since ultrafilters of propositions are just maximal consistent sets of them.
- But so what? Are there anything **wrong** with intensional semantics?

(16) **Some Problematic Aspects of Intensional Semantics**

- The usual reason for rejecting intensional semantics is the endlessly discussed **granularity problem**: as meanings, intensions are too coarse-grained, e.g.
  - the **Hesperus-Phosphorus** problem
  - the **woodchuck-groundhog** problem
  - the **logical omnisicience** problem: if Paris Hilton knows that Paris Hilton is Paris Hilton, then she also knows S, where S is either the Riemann Hypothesis or its denial, whichever is true
- But since I have said all I want to say about the granularity problem already (Pollard 2008a), here I will mention two other problematic aspects:
  - **nonprincipal ultrafilters**
  - **singleton propositions**

(17) **Background for the Nonprincipal Ultrafilters Problem**

- Assume the framework of intensional semantics, where the (**primitive**) **worlds** are the models of "ways things might be".
- Now consider the boolean algebra B of propositions (= the powerset of the set of worlds).
- And next, consider the **ultrafilters** of B (maximal consistent sets of propositions). These appear to **compete** with the worlds to be models of "ways things might be".
- But wait: each primitive world w can be naturally associated with an ultrafilter $\mathsf{u}(w)$, namely the one whose members are the propositions true at w. Isn't this a one-to-one correspondence, so the 'competitors' (to be the **real** ways things might be) come down to the same thing?
- **No!**

(18) **The Nonprincipal Ultrafilters Problem**

- The ultrafilter $\mathsf{u}(w)$ of propositions true at w is **principal**, because it is generated by an atom of B, namely $\{w\}$.
- Uncontroversially, B (the propositions) must be infinite.
- But it is a theorem of ZFC that every infinite boolean algebra has a **non**-principal ultrafilter.
- So let U be one of B's nonprincipal ultrafilters. It is a maximal consistent set of propositions, so **intuitively**, it must correspond to a way things might be.
- But there is no (primitive) world w such that $\mathsf{u}(w) = \mathrm{U}$.
- So U is a maximal consistent set of propositions that for* some mysterious reason doesn't count as a way things might be!
- What went wrong?

(19) **Nonprincipal Ultrafilters in Hyperintensional Semantics**

- Propositions are primitives, worlds are ultrafilters, nobody checks to see whether they are principal or not.
- So the problem does not arise.
- From the point of view of hyperintensional semantics, the nonprincipal ultrafilters problem seems an **artifact** of intensional semantics having made an unfortunate modelling choice.

(20) **Background to the Singular Proposiitions Problem**

- Independent of framework, a **singular** proposition is one which is true at exactly one world.
- In intensional semantics, that means being a singleton set of worlds.
- So let w be a world and let p_w be the singular proposition $\{w\}$.
- It is the conjunction (intersection) of all propositions true at w.
- Since, uncontroversially, there is more than one world, and p_w is only true at w, p_w is **contingent**.

(21) **Some Assumptions about Knowledge**

- For any individual i and any proposition p, we abbreviate by $\mathsf{k}_{i,p}$ the proposition that i knows that p.
- Knowledge is a relation between individuals and propositions.
- Knowledge is (arguably) **factive**: if $\mathsf{k}_{i,p}$ is true at w, then so is p.
- It seems clear that for a given *contingent* proposition p and a given individual i, the proposition $\mathsf{k}_{i,p}$ is also contingent.

(22) **The Omniscience of Paris Hilton**

- But now let $i =$ be Paris Hilton, $w_0 =$ the actual world, $p = p_{w_0}$, and $q = \mathsf{k}_{i,p}$.
- So q is the proposition that Paris Hilton knows the conjunction of every actually true proposition.
- By factivity of knowing, q must be false at every nonactual world.
- But q is contingent, so must be true in the actual world.
- So evidently Paris Hilton is omniscient *simpliciter*!
- This goes beyond Paris Hilton being merely *logically* omniscient (e.g. knowing whether the Riemann hypothesis is true), which (assuming intensional semantics) we have already had to accept.
- For example, one of the conjuncts of p_{w_0} is the one expressed by an English sentence that correctly asserts the number of molybdenum atoms in the Crab Nebula.
- Should semantic theory predict that Paris Hilton knows that?

(23) **Singular Propositions in Hyperintensional Semantics**

- Doubtless, intensional semanticists with a little philosophical training can wiggle out of the Paris Hilton Omniscience problem.
- But from the hyperintensional perspective, that the problem arises at all is another artifact of an unfortunate modelling choice.
- Because, since worlds are just ultrafilters, being a singular proposition is the same thing as being a least member of a principal ultrafilter (easy corollary of Stone's Lemma).
- But for all we know, the propositional preorder is atomless, in which case **none** of its ultrafilters are principal!
- In short, there 's no reason to think there **are** any singular propositions.
- Worrying about whether singular propositions are knowable is a lot like worrying about being gored by a unicorn.

(24) **Hyperintensional Semantics**

- We work in an HOL with separation subtyping and choice.
- Basic types: Prop, Ind, Bool, and Ent.
- For expository simplicity pretend the only type constructor is $\rightarrow$ (actually we also have cartesian product and unit type).
- We axiomatize Prop to be an (internal) boolean preorder:
  - its members model **propositions**
  - its preorder models **entailment**
  - its boolean connectives model the **meanings of the English logic words**
  - its ultrafilters model **worlds**.
- The type World is defined to be $(\text{Prop} \rightarrow \text{Bool})_{\mathsf{ultra}}$, the subtype of Prop $\rightarrow$ Bool restricted by a predicate ultra that expresses internally the property of being an ultrafilter.

(25) **Hyperintensional Types**

- The **hyperintensional** types are obtained by closing the set $\{\text{Prop}, \text{Ind}\}$ under $\rightarrow$ (and other type constructors) and subtyping.
- Members of hyperintensional types are called **hyperintensions**.
- Meanings of NL expressions are modelled by **hyperintensions**.

(26) **Extensional and Intensional Types**

- We define the function Ext from hyperintensional types to types:
  - $\mathrm{Ext}(\mathrm{Prop}) = \mathrm{Bool}$
  - $\mathrm{Ext}(\mathrm{Ind}) = \mathrm{Ent}$
  - $\mathrm{Ext}(A \to B) = A \to \mathrm{Ext}(B)$
  - $\mathrm{Ext}(A_\sigma) = \mathrm{Ext}(A)$
- $\mathrm{Ext}(A)$ is called the **extensional type corresponding to** A.
- We define the function Int from hyperintensional types to types by $\mathrm{Int}(A) = (\mathrm{World} \to \mathrm{Ext}(A))$.
- $\mathrm{Int}(A)$ is called the **intensional type corresponding to** A.

(27) **Extensions of Hyperintensions at Worlds**

a. The theory of extensions is expressed using a family of constants $\mathsf{ext}_A : A \to \text{World} \to \text{Ext}(A)$ (A hyperintensional), often omitting the type-parameter subscript.

b. Note that ext maps each hyperintension to an **intension**.

c. As always, by 'intension' we mean a function from worlds, but in the present setting, the 'worlds' are **ultrafilters of propositions**, not Kripke (1963)/Montague primitive worlds.

d. This is actually very close to **Carnap's** (1947) intensions, since his worlds were 'complete state descriptions'.

e. So we still **have** intensions, we just aren't using them to model meanings.

(28) **Axioms for Extensions of Hyperintensions**

a. For the full set of axioms see Pollard 2008a. For present purposes the two key axioms are for propositions and functions:

b. $\vdash \forall_{p,w}(\mathsf{ext}_{\mathrm{Prop}}(p)(w) = p@w)$
Here '$p@w$' means "p is true at w". [This abbreviates $\mathsf{emb}(w)(p)$, where emb is the constant (supplied by the subtyping schema) that denotes the subtype embedding of World into Prop → Bool.]

c. $\vdash \forall_{w,f}[\mathsf{ext}_{A\to B}(f)(w) = \lambda_x \mathsf{ext}_B(f(x))(w)]$

d. Note that $\mathsf{ext}_{\mathrm{Prop}}$ denotes the Stone embedding of the boolean preorder of propositions into the powerset of the set of ultrafilters: the image of each proposition is its **Stone dual**.

e. The family ext extends Stone duality to all types, that is:
intensions are the Stone duals of hyperintensions.

g. In general, ext is **not** a bijection onto its image. But it becomes so if the axiom is added that makes entailment anisymmetric.

(29) **An Upgrade, not a Competitor**

- Seen in this light, hyperintensional semantics is not a **rival** to intensional semantics, it is a **generalization** of it.
- Worlds and intensions are still there, but so are the intensions' more finally grained duals, the hyperintensions.
- If you're absolutely convinced that entailment is antisymmetric, then you can add the axiom to that effect and then hyperintensions and intensions are in one-to-one correspondence.
- And you still get the benefit of not having to worry about nonprincipal ultrafilters: they are just worlds like any other ultrafilter.
- And if you find singular propositions annoying, you can still banish them by adding the axiom that Prop is atomless.
- 'Tradition' is no reason to keep doing (standard) intensional semantics, because the worlds-as-ultrafilters tradition is older.

(30) **What about Questions?**

- In intensional semantics, the two most influential approaches are due to Karttunen (K, 1997) and to Groenendijk and Stokhof (G, 1984).
- Let's review them and then consider the prospects for generalizing them to the hyperintensional setting.
- The way we will review them is by looking at how they analyze the **polar interrogative**

  Does Fido bark?/whether Fido barks

  and the **consituent interrogative**

  Who barks?

(31) **K (Karttunen) Questions**

- the reference at w of *whether Fido barks* is the singleton set whose member is whichever is true at w, that Fido barks, or that he doesn't
- the reference at w of *which dogs bark* is the set of w-true propositions that x barks, where x is a w-dog.
- In both cases, the reference is a **set of propositions**.
- So the questions themselves have the type World $\rightarrow$ Prop $\rightarrow$ Bool.
- So if we take the hyperintensional type for questions to be Prop $\rightarrow$ Prop or some subtype thereof, their duals would have the type of K-questions.

(32) **G (Groenendijk-Stokhof) Questions**

- The reference at w of *whether Fido barks* is the set of worlds each of which agrees with w about which proposition is true there: that Fido barks, or that he doesn't.
- The reference at w of *which dogs bark* is the set of worlds which agree, for each individual x, about which proposition is true there: that x is a dog that barks, or that it isn't.
- In both cases, the reference is a **set of worlds**.
- So the questions themselves have the type

  $\text{World} \to \text{World} \to \text{Bool}$

  (the type of binary relations on worlds—in fact they are equivalence relations).
- This is **not** dual to any hyperintensional type, so we can't directly generalize G-questions to the hyperintensional setting.

(33) **$K_{\pm}$ (Plus-or-Minus Karttunen) Questions**

- G-questions have one advantage over K-questions: they take **both positive and negative** 'true atomic answers' into account, **for both polar and constituent questions**, while K-questions **throw out the negative answers** in the case of constituent questions.
- Pollard 2008b (following a suggestion by Ken Shan) proposes a modification of K-questions that incorporate this feature.
- Polar questions are handled the same as in K.
- Constituent questions are handled as in K, except **both positive and negative true atomic answers** are kept.
- So the reference of *which dog barked* is the set of all w-true propositions that x barks **or** that x **doesn't** bark, for x a w-dog.

(34) **More on Plus-or-Minus Karttunen Questions**

- Any function induces an equivalance relation on its domain, the relation of being mapped to the same value
- Since $K_{\pm}$-question are of type World $\rightarrow$ Prop $\rightarrow$ Bool, each one induces an equivalence relation on the set of worlds.
- In fact, it is just the corresponding G-question.
- In general, going from a function to the induced equivalence relation losed information: the mapping is not injective.
- For example:

  Which students are vegetarians?

  Whicvh vegetarians are students?

  yield distinct $K_{\pm}$-questions, but the same G-question.
- We will arrange for hyperintensional questions to have have $K_{\pm}$-questions as their duals, but the corresponding G-question can be recovered from the dual if needed.

(35) **Meaning Postulates for *Whether* and *Which***

$\vdash \mathsf{whether'} = \lambda_q \lambda_p [p \ \mathsf{and'} \ ((p \ \mathsf{equals'} \ q) \ \mathsf{or'} \ (p \ \mathsf{equals'} \ \mathsf{not'}(q)))]$

$\vdash \mathsf{which'} = \lambda_P \lambda_Q \lambda_p \mathsf{exists'}(P)(\lambda_x(\mathsf{whether'}(Q(x))(p)))$

Here $x : A$ for A a hyperintensional type; $P, Q : A \rightarrow$ Prop;, and $p, q :$ Prop.

(36) **Two Theorems**

$\vdash \forall_{p,q}[\mathsf{ext}(\mathsf{whether'}(p))(w) = \lambda_q \lambda_p [p@w \wedge ((p = q) \ \vee (p = \mathsf{not'}(q)))]]$

$\vdash \forall_{P,Q,w}[\mathsf{ext}(\mathsf{which'}(P)(Q))(w) = \lambda_p \exists_x [P(x)@w \wedge p@w \wedge ((p = Q(x)) \vee (p = \mathsf{not'}(Q(x))))]]$

(37) **Two Hyperintensional Questions and their Extensions**

a. whether Fido barks

$\lambda_p[p$ and' $((p$ equals' bark'(Fido') or' $(p$ equals' not'(bark'(Fido)))))]

$\lambda_p[p@w \wedge ((p = \text{bark'(Fido')}) \vee (p = \text{not'(bark'(Fido))}))]$

b. Which dogs bark?

$\lambda_p$exists'(dog')$(\lambda_x$(whether'(bark'$(x))(p)))$

$\lambda_p \exists_x(\text{dog'}(x)@w \wedge p@w \wedge ((p = \text{bark'}(x)) \vee (p = \text{not'(bark'}(x)))))$

(38) **Hyperintensional Questions vs. their Duals**

- In general, unless entailment is antisymmetric, the mapping from hyperintensions to their duals is not injective.
- For example:

  whether Paris Hilton is Paris Hilton

  whether R [where R asserts the Riemann Hypothesis]

  are distinct hyperintensional questions, but the same (plus-or-minus) Karttunen question.
- So hyperintensional questions are a natural, more fine-grained generalization of plus-or-minus Karttunen questions, from which G-questions can also be straightforwardly recovered.

(39) **Summary of Hyperintensional Semantics**

- It is an upgrade/repair on intensional semantics.
- It eschews primitive worlds, and reverts to the earlier Jónsson/Tarski treament where worlds are ultrafilters of propositions.
- There are still intensions, but they are the Stone duals of the meanings, not the meanings themselves.
- Artifactual problems with intensional semantics (obligatory anti-symmetry of entailment, obligatory singular propositions, maximal consistent sets of propositions that aren't worlds) vanish.
- We considered the consequences of 'going hyper' for questions.
- The hyperintensional questions have (improved) K-questions as their duals, provide a simple reconstruction of G-questions, and are more fine-grained than either.