PROBLEM SET FOUR: THE NATURAL NUMBERS

Problem 1

Prove the theorem that ω is inductive.

Problem 2

Prove the theorem that ω is a subset of every inductive set.

Problem 3

Prove PMI.

Problem 4

Use PMI and the definition of $+$ to prove that for every natural number n, $\text{suc}(n) = 1 + n$.

Problem 5

Use PMI and the definition of $\cdot$ to show that for every natural number n, $1 \cdot n = n$.

Problem 6

Define the exponentiation operation $\star$, where $m \star n$ is the number customarily written m^n. [Hint: as with addition and multiplication, start out by holding m fixed. The heart of the question is correctly identifying the appropriate values of X, x, and F to use in order to apply the Recursion Theorem.]

Problem 7

Draw Hasse diagrams to show that it is possible for:

a. an order to have a unique maximal element but no top;

b. an order to have more than one maximal element but no top;

c. a preorder to have more than one top;

d. an order to have no maximal element.

Problem 8

Draw a Hasse diagram for the subset-inclusion order on the set $\wp(3)$.