Mathese: Spoken Predicate Logic

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These slides are available at:

`http://www.ling.osu.edu/~scott/680`

(1) **And**

- The standard abbreviation for *and* is the symbol $\wedge$, called **conjunction**.
- *And* is used for combining sentences to form a new sentence:

  S_1 and S_2. (Abbreviated form: $S_1 \wedge S_2$)
- A sentence formed this way is called a **conjunctive** sentence.
- Here S_1 is called the **first conjunct** and S_2 is called the **second conjunct**.
- A conjunctive sentence is considered to be true if both conjuncts are true. Otherwise it is false.

(2) **Or**

- The standard abbreviation for *or* is the symbol $\vee$, called **disjunction**.
- *Or* is used for combining sentences to form a new sentence:

  S_1 or S_2. (Abbreviated form: $S_1 \vee S_2$)
- A sentence formed this way is called a **disjunctive** sentence.
- Here S_1 is called the **first disjunct** and S_2 is called the **second disjunct**.
- A disjunctive sentence is considered to be true if at least one disjunct is true. Otherwise it is false.

(3) **Implies**

- The standard abbreviation for *implies* is the symbol $\rightarrow$, called **implication**.
- Some authors write $\supset$ instead of $\rightarrow$ for implication.
- *Implies* is used for combining sentences to form a new sentence:

  S_1 implies S_2. (Abbreviated form: $S_1 \rightarrow S_2$)
- A synonym for 'implies' is 'if ..., then ...', as in:
  If S_1, then S_2.
- A sentence formed this way is called an **implicative** sentence, or alternatively, a **conditional** sentence.
- Here S_1 is called the **antecedent** and S_2 is called the **consequent**.

- A conditional sentence is considered to be true if either the antecedent is false or the consequent is true (or both), *even if the antecedent and the consequent seem to have nothing to do with each other*. Otherwise it is false.
- For example:

  If there does not exist a set with no members, then $0 = 0$.

  is true!
- Another example:

  If $0 \neq 0$ then $1 \neq 1$.

  is true!

(4) **If and only if**

- The standard abbreviation for *if and only if* is the symbol $\leftrightarrow$, called **biimplication**.
- A synonym for *if and only if* is the invented word *iff*.
- *If and only if (iff)* is used for combining sentences to form a new sentence:

  S_1 iff S_2. (Abbreviated form: $S_1 \leftrightarrow S_2$)

- A sentence formed this way is called an **biimplicative** sentence, or alternatively, a **biconditional** sentence.

- A biconditional sentence is considered to be true if either (1) both S_1 and S_2 are true, or (2) both S_1 and S_2 are false. Otherwise, it is false.
- S_1 iff S_2 can be thought of as shorthand for:

  S_1 implies S_2, and S_2 implies S_1.

(5) **It is not the case that (1/3)**

- The standard abbreviation for *it is not the case that* is the symbol ¬, called **negation**.
- Some authors write ∼ instead of ¬ for negation.
- Negation is written before the sentence it negates:

  It is not the case that S. (Abbreviated form: ¬S)
- The sentence *it is not the case that S* is called the **negation** of S, or, equivalently, the **denial** of S, and S is called the **scope** of the negation.
- A sentence formed this way is called a **negative** sentence.
- More colloquial synonyms of *it is not the case that S* are *S not!* and *no way S*.
- Unsurprisingly, a negative sentence is considered to be true if the scope is false, and false if the scope is true.

(6) **It is not the case that (2/3)**

- Often, the effect of negation with *it is not the case that* can be achieved by ordinary English **verb negation**, which involves:
  - replacing the finite verb (the one that agrees with the subject) V with 'does not V' if V is not an auxiliary verb (such as *has* or *is*), or
  - negating V with a following *not* or *-n't* if it *is* an auxiliary.
- for example, these pairs of sentences are equivalent (express the same thing):

  It is not the case that 2 belongs to 1.

  2 does not belong to 1.

  It is not the case that 1 is empty.

  1 isn't empty.

(7) **It is not the case that (3/3)**

- But: negation by *it is not the case that* and verb negation cannot be counted on to produce equivalent effects if the verb is in the scope of a *quantifier* (see below).
- Example: these are not equivalent:

  (i) It is not the case that for every x, x belongs to x.

  (ii) For every x, x doesn't belong to x.
- For (i) is clearly true (for example, 0 doesn't belong to 0). But the truth or falsity of (ii) can't be determined on the basis of the assumptions about sets made in Chapter 1. (In fact, different ways of adding further set-theoretic assumptions resolve the issue in different ways.)

(8) **Variables**

- Roughly speaking, Mathese variables are the counterparts of ordinary English pronouns (but without such distinctions as case, number, and gender).
- Variables are "spelled" as upper- or lower-case roman letters (usually italicized except in handwriting), with or without numerical subscripts, e.g. x, y, x_0, x_1, X, Y, etc.
- In a context where the subject matter is set theory, we think of variables as ranging over arbitrary sets.