

PROBLEM SET EIGHT: PROPOSITIONAL LOGIC

Problem 1

Make a truth table for the formula

$$((A \vee B) \rightarrow C) \rightarrow ((A \rightarrow B) \wedge (B \rightarrow C))$$

and then tell whether the formula is valid.

Problem 2

Remember that in the standard semantics of PL, $\text{Prop} = \wp(V)$, where V is the set of valuations, and for any formula ϕ , $\text{sem}(\phi)$ is the set of valuations that satisfy ϕ (i.e. that assign ϕ the value 1 (true)). Recall also that in this semantics, entailment is just the \subseteq relation on Prop . Answer the following questions about sem , with reference to operations on sets:

- What is $\text{sem}(T)$?
- What is $\text{sem}(F)$?
- What is $\text{sem}(\sim \phi)$ in terms of $\text{sem}(\phi)$?
- What is $\text{sem}(\phi \wedge \psi)$ in terms of $\text{sem}(\phi)$ and $\text{sem}(\psi)$?
- What is $\text{sem}(\phi \vee \psi)$ in terms of $\text{sem}(\phi)$ and $\text{sem}(\psi)$?
- What is $\text{sem}(\phi \rightarrow \psi)$ in terms of $\text{sem}(\phi)$ and $\text{sem}(\psi)$?
- What is $\text{sem}(\phi \leftrightarrow \psi)$ in terms of $\text{sem}(\phi)$ and $\text{sem}(\psi)$?

Problem 3

As we discussed in class, there is not universal agreement about what the truth value of an English sentence of the form *if S then S'* is, given the truth values of S and S'.

Explain as clearly as you can what you think the answer to this question is. There is no right answer, at least not so far, but the point of the question is not to give a definitive answer but to get as clear as possible what your own view is.

Problem 4

Prove the three facts about rpc operations (Un-Curry, Antitonicity on First Argument, Monotonicity on Second Argument).

Problem 5

Prove the seven facts about HPS's at the end of Chapter 9, section 4.

Problem 6

After reviewing Chapter 10, section 4, derive the following two rules. Remember that deriving a rule is like giving a formal proof, except that the leaves can be labelled with premisses of the rule to be derived, not only with axioms.

(HS) Hypothetical Syllogism:

$$\frac{\phi \vdash \psi \quad \psi \vdash \psi}{\phi \vdash \psi}$$

Converse of Implication Introduction:

$$\frac{\Gamma \vdash \phi \rightarrow \psi}{\Gamma, \phi \vdash \psi}$$

Problem 7

Give formal proofs of the following sequents (all of which have empty contexts). Remember that you can use any rules that have already been derived, and that a leaf can be labelled not only by an axiom, but also by any sequent that has already been shown to be provable.

- a. $\vdash \phi \rightarrow T$
- b. $\vdash (\phi \wedge \psi) \rightarrow \psi$
- c. $\vdash (\phi \wedge \psi) \rightarrow (\psi \wedge \phi)$
- d. $\vdash \phi \rightarrow (\phi \wedge \phi)$
- e. $\vdash ((\phi \rightarrow \psi) \wedge \phi) \rightarrow \psi$