PROBLEM SET FIVE: INFINITIES

Problem 1

Prove the theorem that no natural number is Dedekind infinite.

Problem 2

Prove the corollary that no finite set is Dedekind infinite.

Problem 3

Prove the corollary that any Dedekind infinite set is infinite.

Problem 4

Prove the corollary that no two distinct natural numbers are equinumerous.

Problem 5

Prove the corollary that for any finite set A, there is a unique natural number equinumerous with A.

Problem 6

Prove the lemma that if $C \subsetneq n \in \omega$, then $C \approx m$ for some $m < n$.

Problem 7

Prove that for any sets A, B, and C:

a. $A \preceq A$;

b. if $A \preceq B$ and $B \preceq C$ then $A \preceq C$; and

c. $A \preceq \wp(A)$.

Problem 8

Prove the corollary that any countably infinite set is equinumerous with ω.

Problem 9

Prove the corollary that any infinite subset of ω is equinumerous with ω.

Problem 10

Prove that $\wp(\omega)$ is nondenumerable.

Problem 11

Show how the Recursion Theorem justifies the definition of the function C used in the proof (in the Appendix below) of the Schröder-Bernstein Theorem.

Problem 12

Show how the Recursion Theorem justifies the definition of the function h used in the proof of the theorem (in the Appendix below) that every infinite set dominates ω.