

## PROBLEM SET FOUR: THE NATURAL NUMBERS

### Problem 1

Prove the theorem that  $\omega$  is inductive.

### Problem 2

Prove the theorem that  $\omega$  is a subset of every inductive set.

### Problem 3

Prove PMI.

### Problem 4

Use PMI and the definition of  $+$  to prove that for every natural number  $n$ ,  $\text{suc}(n) = 1 + n$ .

### Problem 5

Use PMI and the definition of  $\cdot$  to show that for every natural number  $n$ ,  $1 \cdot n = n$ .

### Problem 6

Define the exponentiation operation  $\star$  (see end of subsection 3.2), where  $m \star n$  is the number customarily written  $m^n$ . [Hint: as with addition and multiplication, start out by holding  $m$  fixed. What I am really after here is correctly identifying the appropriate values of  $X$ ,  $x$ , and  $F$  to use in order to apply the Recursion Theorem.]

### Problem 7

Draw Hasse diagrams to show that it is possible for:

- an order to have a unique maximal element but no top;
- an order to have more than one maximal element but no top;
- a preorder to have more than one top;
- an order to have no maximal element.

**Problem 8**

Draw a Hasse diagram for the subset-inclusion order on the set  $\wp(\mathbb{3})$ .