PROBLEM SET TWO: CARTESIAN PRODUCT AND COPRODUCT; MATHESE

Problem 1

Prove that for any set a, $a \times \emptyset = \emptyset$.

Problem 2

Use a Separation assumption to prove that for any sets a and b, there exists a unique set which is the cartesian product $a \times b$. [Hint: the hard part is figuring out which set to use as the one that the members to be collected into $a \times b$ are separated out from.]

Problem 3

Use a Separation assumption to show that for any sets a and b, there exists a unique set which is the cartesian coproduct $a + b$. [Hint: the hard part is figuring out which set to use as the one that the members to be collected into $a + b$ are separated out from.]

Problem 4

Translate the following Mathese sentences into Mathese symbols. Don't worry about whether the sentences are true or false, or provable or unprovable; in math it's necessary to be able to express things that are false, and things that are true but unprovable.

a. There exists a set x such that [(the empty set is in x) and (for every set y, if y is in x then the successor of y is in x)].

b. There does not exist a set x such that [(x is a member of x) and (for every set y, if y is in x then y is equal to x)].

c. For every set n, n belongs to the successor of n.

d. For every set n, either n is zero or there exists a set m such that n is the successor of m.

Problem 5

Translate the Mathese sentences in Problem 4 into sentences of ordinary English.

Problem 6

Translate the following sequences of symbols into clear, unambiguous English, either standard English or Mathese or a mixture of the two, whichever you prefer. Again, don't worry about whether the sentences are true, or whether they are provable. [*Note*: In Mathese, there is a standard way of avoiding repeating sequences of quantifiers of the same kind (i.e. all universal or all existential), e.g.:

instead of "For every x, for every y ...", say "For all x and y ..." or "For any two sets x and y, ..."

instead of "There exists x such that there exists y such that there exists z ...", say "There exist three sets x, y, and z such that ...'.]

a. $\forall x \exists y \forall z (z \in y \leftrightarrow \ \exists u \exists v [z \in u \wedge u \in v \wedge v \in x])$

b. $\exists ! x \forall y (y \in x \leftrightarrow \ y \notin y)$

c. $\forall x \forall y [x \neq y \rightarrow (s(x) \neq s(y))]$

d. $\forall x ([0 \in x \wedge \forall y (y \in x \rightarrow s(y) \in x)] \rightarrow z \subseteq x)$