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Cross-linguistic variation in temporal adjunct clauses

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Goal: In this talk we propose a *semantic* analysis of cross-linguistic variation in the interpretation of embedded tenses in Temporal Adjunct Clauses (TACs), and argue that it is empirically and theoretically more adequate than a *syntactic* analysis.

Outline:

§1 Cross-linguistic variation in TACs in English, Japanese and Russian

§2 Previous analyses of individual languages and the cross-linguistic variation

§3 Our semantic analysis

§4 Wider implications of our analysis

1 TACs[1] in English, Japanese and Russian

1.1 Tense in matrix clauses

English: past versus non-past tense.

(1) **Past**
Anna sang yesterday/#now/#tomorrow.

(2) **Non-past**
Anna sings now/tomorrow.

Japanese: past versus non-past tense.

(3) **Past**

Anna-wa kinoo **hasit-ta**.
Anna-TOP yesterday run-PAST

'Anna ran yesterday.'

[1]We are restricting ourselves to *before*-clauses here to illustrate the cross-linguistic variation but the analysis in the Appendix accounts for *after*-clauses, too.

(4) **Non-past**

Anna-wa asita/#ima **hasi-ru**.
Anna-TOP tomorrow/now run-NPST

'Anna will run tomorrow/#Anna is running now.' [2]

Russian: past versus non-past tense.

(5) **Past**

Anna včera **pe-l-a**.
Anna yesterday sing-PAST-FEM

'Anna sang yesterday.'

(6) **Non-Past**

Anna sejčas/zavtra **poj-ot**.
Anna now/tomorrow sing-NPST

'Anna is singing now./Anna will sing tomorrow.'

Question: How are these tenses interpreted in embedded clauses?

1.2 Tenses in TACs

English:

(7) a. Anna left before Ken arrived.
b. #Anna left before Ken arrives.

Japanese:

(8) a. [Ken-ga **ku-ru** mae-ni] Anna-ga kaet-ta.
Ken-NOM arrive-NPST before-at Anna-NOM leave-PAST
'Anna left before Ken arrived.'
b. #[Ken-ga **ki-ta** mae-ni] Anna-ga kaet-ta.
Ken-NOM arrive-PAST before-at Anna-NOM leave-PAST
'Anna left before Ken arrived.'

[2]The 'now' sentence is actually fine with the reading 'Anna is about to run.'

Russian:

(9) a. Anna u-exa-l-a [pered tem, kak Ken
Anna PERF-leave-PAST-FEM before that.INSTR as Ken
pri-exa-l].
PERF-arrive-PAST.MASC
'Anna left before Ken arrived.'

b. #Anna u-exa-l-a [pered tem, kak Ken **pri-ed-et**].
Anna PERF-leave-PAST-FEM before that.INSTR as Ken PERF-arrive-NPST
Intended: 'Anna left before Ken arrived.'

1.3 Summary

	matrix clause tenses	With a past tense matrix clause the *before*-TACs has *x* tense.
English	past, non-past	**past**
Japanese	past, non-past	non-past
Russian	past, non-past	**past**

Table 1: TACs in English, Japanese and Russian

Empirical generalization: English and Russian behave alike in TACs (in contrast to Japanese).

Question: How can this cross-linguistic variation in the distribution of embedded tense be accounted for, especially in light of the distribution of tense in other embedded environments, such as Propositional Attitude Complements (PACs)?

2 Previous analyses

2.1 Ogihara's (1994) analysis of TACs in English and Japanese

Ogihara accounts for the distribution of tenses in terms of the Sequence of Tense (SOT) rule. This rule is formulated as a parameter, which divides languages into two typological classes, SOT langauges (English) and non-SOT langauges (Japanese).

Semantic effect of the SOT rule in English PACs:

(10) Ken said that Anna was sick.
(backward shifted reading & overlapping reading)

The SOT rule accounts for the overlapping reading:

(11) **LF of English PAC** in (10):
Ken say-PAST [that Anna be-~~PAST~~ sick]

Temporal interpretation of TACs:

- **Assumption 1**: Embedded tenses in TACs in all languages are interpreted relative to the matrix Event Time (ET), not to the Speech Time (ST).
- **Assumption 2**: The temporal order between the matrix event and the embedded event is established by the meaning of temporal connectives (e.g. 'before' requires the matrix event to temporally precede the embedded event).

Japanese:

- Temporal interpretation of Japanese TACs can be accounted for in terms of the above two assumptions.

(12) a. [Ken-ga **ku-ru** mae-ni] Anna-ga kaet-ta.
Ken-NOM arrive-NPST before-at Anna-NOM leave-PAST
'Anna left before Ken arrived.'

b. **LF representation:**
[Ken arrive-NPST before] Anna leave-PAST
embedded NPST: Anna leave < Ken arrive
'before': Anna leave < Ken arrive

(13) a. #[Ken-ga **ki-ta** mae-ni] Anna-ga kaet-ta.
Ken-NOM arrive-PAST before-at Anna-NOM leave-PAST
'Anna left before Ken arrived.'

b. **LF representation:**
#[Ken arrive-PAST before] Anna leave-PAST
embedded PAST: Ken arrive < Anna leave
'before': Anna leave < Ken arrive

English:

- However, the temporal interpretation of Enlish TACs cannot be accounted for with the above two assumptions alone.

(14) Anna left before Ken arrived.

- **Assumption 3**: In order to account for the order between the two events, Ogihara proposes that the (SOT) rule applies in TACs as well and deletes the embedded tense at LF.

(15) **LF of English TAC** in (14):
Anna leave-PAST [before Ken arrive-~~PAST~~]
'before': Anna leave < Ken arrive

Problems with Ogihara's analysis:

As noted by Arregui and Kusumoto (1998) (henceforth A&K), languages like Polish, where the interpretation of propositional attitude clauses (PACs) motivates no SOT rule, are problematic for this analysis (we have given comparable data from Russian).[3]

(16) a. Anna u-exa-l-a [pered tem, kak Ken
Anna PERF-leave-PAST-FEM before that.INSTR as Ken
pri-exa-l].
PERF-arrive-PAST.MASC
'Anna left before Ken arrived.'

b. **LF representation:**
Anna leave-PAST [before Ken arrive-PAST before]
embedded PAST: Ken arrive < Anna leave
'before': Anna leave < Ken arrive

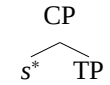
2.2 Arregui and Kusumoto's (1998) analysis of TACs in English, Japanese and Polish

Arregui and Kusumoto reject Ogihara's SOT-based analysis of the TAC variation and propose a syntactic solution for the contrast between English/Polish TACs and

[3]Kondrashova (2005) suggests that Russian should be analyzed as a Split-SOT language, but it was not regarded as an option in A&K.

Japanese TACs. Unlike Ogihara, they assume that whether an embedded tense is interpreted with respect to the matrix ET is a matter of syntactic selection.

- **Assumption 1**: The ST is overtly realized in the head of CP, but TP does not contain the ST.



- **Assumption 2**: Temporal connectives in different languages select for different syntactic structures: English/Russian 'before' select for a CP, so the embedded tense is interpreted with respect to the ST, while Japanese 'before' selects for a TP, and the embedded tense is interpreted with respect to the matrix clause ET.

(17) English/Russian:
Anna leave-PAST before [**CP** Ken arrive-PAST]
(evaluated relative to the **ST**)

(18) Japanese:
#Anna leave-PAST before [**TP** Ken arrive-PAST]
(evaluated relative to the **matrix ET**)

2.3 Summary

- **Ogihara** accounts for the temporal interpretation of English/Japanese TACs in terms of the (non-) application of the SOT rule. However, the SOT-based analysis cannot account for languages like Russian.

	PACs	TACs
English	SOT	SOT
Japanese	no SOT	no SOT
impossible	no SOT	SOT

Table 2: Typological predictions of Ogihara's analysis

- **A&K** account for the temporal interpretation of English/Japanese/Polish(Russian) TACs in terms of their different syntactic structures. However, their analysis involves unmotivated complications such as
  - cross-linguistically different structures of TACs (i.e. TP or CP);
  - non-uniform tense meanings (i.e. past tense is an operator, present tense is a variable);
  - difference in the meaning of temporal connectives.

3 A semantic analysis

General methodological principles:

- We take the principle of compositionality seriously.
- We assume uniform interpretation of tense meanings in matrix and embedded clauses.

Proposal about TACs:

Cross-linguistic variation in the tense interpretation in TACs is given a **semantic** account in terms of the differences in the **meanings** of tempral connectives (across languages).

3.1 Basic assumptions about temporal interpretation

(19) Ken arrived.

Mechanics of compositional semantics[4]

- (Untensed) sentence radicals (e.g. Ken arrive) denote sets of times (temporal abstracts; of type $\langle i,t\rangle$):

  (20) $\textsf{Ken arrive} \Rightarrow \lambda t[\mathrm{AT}(t, \mathit{arrive}'(k))]$

- Tenses are temporal modifiers of type $\langle\langle i,t\rangle,\langle i,t\rangle\rangle$:

  (21) $\textsc{past} \Rightarrow \lambda P_{\langle i,t\rangle}\lambda t[P(t) \wedge \mathbf{past}(t)]$

- Tensed sentences are again temporal abstracts (i.e. sets of times):

  (22) $\textsc{past}(\textsf{Ken arrive}) \Rightarrow \lambda P_{\langle i,t\rangle}\lambda t[P(t) \wedge \mathbf{past}(t)](\lambda t[\mathrm{AT}(t, \mathit{arrive}'(k))])$
  $= \lambda t[\mathrm{AT}(t, \mathit{arrive}'(k)) \wedge \mathbf{past}(t)]$

- We assume that **existential closure** applies to matrix clauses (at the level of discourse):

  (23) (22) after existential closure:
  $\exists t[\mathrm{AT}(t, \mathit{arrive}'(k)) \wedge \mathbf{past}(t)]$

[4] We follow Dowty (1979), Stump (1985) and Yoon (1996) in these basic assumptions.

How tenses are *evaluated*

- Tenses locate event times relative to the **local evaluation time**,[5] which is the speech time in matrix clauses (see the next bullet point).

  (24) $[\![\mathbf{past}(\zeta)]\!]^{M,w,i,g} = 1$ iff $[\![\zeta]\!]^{M,w,i,g} < i$

  (25) $[\![\mathrm{AT}(\zeta, P)]\!]^{M,w,i,g} = 1$ iff $[\![P]\!]^{M,w,i',g} = 1$ where $i' = [\![\zeta]\!]^{M,w,i,g}$

- Matrix clauses are interpreted with respect to the speech time s^* (and the actual world w_0):

  (26) $[\![\exists t[\mathrm{AT}(t, \mathit{arrive}'(k)) \wedge \mathbf{past}(t)]]\!]^{M,w_0,s^*,g} = 1$ iff there is some time t prior to the speech time and Ken arrives at t.

- For embedded tenses, the evaluation time is a time provided by the embedding construction (and this time need not be the speech time).

3.2 Temporal Adjunct Clauses (TACs)

Reminder of data

English and Russian pattern alike, in contrast to Japanese.

- Tense in TACs is evaluated relative to the **matrix event time** in Japanese.
- Tense in TACs is evaluated relative to the **speech time** in English and Russian.

Japanese:

(27) [Ken-ga **ku-ru** mae-ni] Anna-ga kaet-ta.
Ken-NOM arrive-NPST before-at Anna-NOM leave-PAST
'Anna left before Ken arrived.'

Ogihara's analysis translated into our system:

(28) a. $\textsf{Anna leave-}\textsc{past} \Rightarrow \lambda t[\mathrm{AT}(t, \mathit{arrive}'(k)) \wedge \mathbf{past}(t)]$
(matrix tense, interpreted w.r.t. s^*)

b. $\textsf{Ken arrive-}\textsc{npst} \Rightarrow \lambda t[\mathrm{AT}(t, \mathit{arrive}'(k)) \wedge \mathbf{npst}(t)]$
(embedded tense, interpreted w.r.t. evaluation time = matrix ET)

[5] In our technical implementation, the evaluation time is not part of the object language but is kept track of as a temporal parameter that is fixed when logical expressions receive model-theoretic interpretations. However, nothing hinges on this assumption. The analysis could be formulated equally well in a system in which the evaluation time was part of the object language, such as, e.g., the fragment of Ogihara (1996).

c. mae 'before' $\Rightarrow \lambda P\lambda Q\lambda t[Q(t) \wedge \mathbf{AT}(\boldsymbol{t}, \exists t_1[\underline{P(t_1)} \wedge t < t_1])]$

evaluation time of embedded clause = **matrix ET**

d. $\exists t[\mathbf{past}(t) \wedge \mathrm{AT}(t, \mathit{leave'}(a)) \wedge \mathrm{AT}(t, \exists t_1[\underline{\mathbf{npst}(t_1) \wedge \mathrm{AT}(t_1, \mathit{arrive'}(k))} \wedge t < t_1])]$
"There is a time t that is in the past of the speech time s^* and at which Anna leaves and at which there is a non-past time t_1 ($t < t_1$) at which Ken arrives and t precedes t_1 ($t < t_1$)."

e.

Accounts for the infelicity of (8b):

(29) a. #[Ken-ga **ki-ta** mae-ni] Anna-ga kaet-ta. (= (8b))
Ken-NOM arrive-PAST before-at Anna-NOM leave-PAST
'Anna left before Ken arrived.'

b. $\exists t[\mathbf{past}(t) \wedge \mathrm{AT}(t, \mathit{leave'}(a)) \wedge \mathrm{AT}(t, \exists t_1[\mathbf{past}(t_1) \wedge \mathrm{AT}(t_1, \mathit{arrive'}(k)) \wedge t < t_1])]$
"There is a time t that is in the past of the speech time s^* and at which Anna leaves and at which there is a past time t_1 ($t_1 < t$) at which Ken arrives and t precedes t_1 ($t < t_1$)."

English and Russian:

But the English/Russian examples corresponding to (8b)/(29a) are felicitous!

Proposal: The evaluation time of the embedded clause is identified with the evaluation time of the matrix clause.

$\Rightarrow$ We depart from Ogihara in this respect.

- Specifically, we reject Ogihara's **Assumption 1**.
- But we maintain his **Assumption 2**, thus fully retaining the semantically-based aspect of his analysis.

(30) English before/Russian pered 'before' $\Rightarrow \lambda P\lambda Q\lambda t[\exists t_1(Q(t) \wedge \underline{P(t_1)} \wedge t < t_1)]$

evaluation time of embedded clause = **evaluation time of matrix clause** ($= s^*$)

(31) Anna left before Ken arrived.

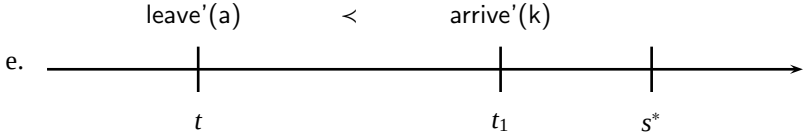
a. Anna leave-PAST $\Rightarrow \lambda t[\mathrm{AT}(t, \mathit{leave'}(a)) \wedge \mathbf{past}(t)]$
(matrix tense, interpreted w.r.t. s^*)

b. Ken arrive-PAST $\Rightarrow \lambda t[\mathrm{AT}(t, \mathit{arrive'}(k)) \wedge \mathbf{past}(t)]$
(embedded tense, interpreted w.r.t. evaluation time = s^*)

c. before $\Rightarrow \lambda P\lambda Q\lambda t[\exists t_1[Q(t) \wedge \underline{P(t_1)} \wedge t < t_1]]$

d. $\exists t\exists t_1[\mathbf{past}(t) \wedge \mathrm{AT}(t, \mathit{leave'}(a)) \wedge \underline{\mathbf{past}(t_1) \wedge \mathrm{AT}(t_1, \mathit{arrive'}(k))} \wedge t < t_1]$
"There is a time t that is in the past of the speech time s^* ($t < s^*$) and at which Anna leaves and there is a time t_1 that is in the past of the speech time ($t_1 < s^*$) and at which Ken arrives, and t precedes t_1 ($t < t_1$)."

e.



4 Conclusions and wider implications

Summary of the present proposal

- The cross-linguistic variation in TACs is accounted for *semantically* as variation in the meanings of temporal connectives.
- The key notion by means of which the cross-linguistic variation is accounted for is the **evaluation time** of clauses.
  - 'Before' in Japanese sets the evaluation time of the embedded clause to the **matrix event time**.
  - 'Before' in English and Russian sets the evaluation time of the embedded clause to the **speech time**.
- Our semantic analysis dispenses with complicated syntactic mechanisms such as:
  - the SOT rule in Ogihara (1994, 1996)
  - syntactic correlates of inherently semantic notions such as the speech time variable s^* in the functional head C(omp) in A&K

Questions for future research

- Can we extend this semantically-based analysis to propositional attitude complements and relative clauses (regarding a possible extension to PACs, cf. Kubota et al. 2008)?
- How can the present analysis be extended to take into account the interactions between tense and aspect?

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A Ogihara's (1994, 1996) objections to Stump's (1985) absolute-tense analysis of TACs in English

Ogihara actually does consider a possible alternative to his analysis of the English/Japanese variation in TACs which is based on Stump's (1985) analysis of English TACs and which is very close to our proposal. We reproduce here the reasons for which he dismisses that alternative together with our own responses to them.

Reason 1: The analysis does not account for the fact that the TAC cannot be in the future tense (Ogihara 1994:225).

(32) *John will call Mary when he will finish his book.

Response: No categorical restriction against future tense TACs.

As noted by Ogihara (1994) himself in footnote 7, Smith (1975:73) points out that cases involving verb phrase deletion are acceptable:

(33) John will leave when Mary will.

Reason 2: The analysis overgenerates an unavailable reading when the matrix clause is in future tense (Ogihara 1994:255, Ogihara 1996:184f.)

(34) a. John will call Mary after he finishes his assignment.
b. John will call Mary before he finishes his assignment.
(Ogihara 1994:225)

If present tense has non-past meaning, (34a) would mean that John's finishing happens at the speech time.

Response: This interpretation actually doesn't arise.

(i) Remember: Simple present tense events overlap with speech time only if interpreted habitually; if episodic, they receive a future time interpretation.

(2) a. Anna is in her office now/tomorrow. (state: present/future)
b. Anna sings now/tomorrow.
(event: habitual present/scheduled futurate)

(ii) TACs accept only episodically-interpreted eventualities:

(35) a. John believes Mary.
b. John will be happy after he believes Mary.
(coerced inceptive interpretation)

(36) a. John knows Mary.
b.??John will be happy after he knows Mary.

(37) a. John finishes his assignment.
(habitual present/scheduled futurate)
b. John will call Mary after he finishes his assignment.
(futurate only)

Since only events are felicitous as TACs, there is no danger that a present time interpretation would arise with present tense TACs.

B Fragment

B.1 Translations from quasi-English/Japanese/Russian into IL

(38) Ken arrive $\Rightarrow \lambda t[\mathrm{AT}(t, \mathit{arrive}'(k))]$

(39) a. PAST $\Rightarrow \lambda P_{\langle i,t\rangle}\lambda t[P(t) \wedge \mathbf{past}(t)]$

b. NPST $\Rightarrow \lambda P_{\langle i,t\rangle}\lambda t[P(t) \wedge \mathbf{npst}(t)]$

(40) a. mae-ni (Japanese 'before') $\Rightarrow \lambda P\lambda Q\lambda t[Q(t) \wedge \mathrm{AT}(t, \exists t_1[\underline{P(t_1)} \wedge t < t_1])]$

b. before, pered (Russian 'before') $\Rightarrow \lambda P\lambda Q\lambda t[\exists t_1[Q(t) \wedge \underline{P(t_1)} \wedge t < t_1]]$

(41) a. ato-de (Japanese 'after') $\Rightarrow \lambda P\lambda Q\lambda t[Q(t) \wedge \mathrm{AT}(t, \exists t_1[\underline{P(t_1)} \wedge t_1 < t])]$

b. after, posle (Russian 'after') $\Rightarrow \lambda P\lambda Q\lambda t[\exists t_1[Q(t) \wedge \underline{P(t_1)} \wedge t_1 < t]]$

B.2 Model theoretic interpretations of IL expressions

(42) a. $[\![\mathbf{past}(\zeta)]\!]^{M,w,i,g} = 1$ iff $[\![\zeta]\!]^{M,w,i,g} < i$

b. $[\![\mathbf{npst}(\zeta)]\!]^{M,w,i,g} = 1$ iff $i \leq [\![\zeta]\!]^{M,w,i,g}$

(43) $[\![\mathrm{AT}(\zeta, P)]\!]^{M,w,i,g} = 1$ iff $[\![P]\!]^{M,w,i',g} = 1$ where $i' = [\![\zeta]\!]^{M,w,i,g}$

B.3 Sample derivation for a Japanese TAC

(44) a. (Japanese):
Anna leave-PAST before Ken arrive-PAST $\Rightarrow$
$\lambda t[\mathbf{past}(t) \wedge \mathrm{AT}(t, \mathit{leave}'(a)) \wedge \mathbf{AT}(t, \exists t_1[\mathbf{npst}(t_1) \wedge \mathrm{AT}(t_1, \mathit{arrive}'(k)) \wedge t < t_1])]$

b. $\exists t[\mathbf{past}(t) \wedge \mathrm{AT}(t, \mathit{leave}'(a)) \wedge \mathbf{AT}(t, \exists t_1[\mathbf{npst}(t_1) \wedge \mathrm{AT}(t_1, \mathit{arrive}'(k)) \wedge t < t_1])$]
(existential closure)

(45) $[\![(44b)]\!]^{M,w,s^*,g} = 1$ iff there is some g' such that $g' = g$ (except possibly that $g'(t) \neq g(t)$) and
$[\![\mathbf{past}(t) \wedge \mathrm{AT}(t, \mathit{leave}'(a)) \wedge \underline{\mathrm{AT}(t, \exists t_1[\mathbf{npst}(t_1) \wedge \mathrm{AT}(t_1, \mathit{arrive}'(k)) \wedge t < t_1])}]\!]^{M,w,s^*,g'} = 1$

(46) (underlined part of (45):)
$[\![\mathrm{AT}(t, \exists t_1[\mathbf{npst}(t_1) \wedge \mathrm{AT}(t_1, \mathit{arrive}'(k)) \wedge t < t_1])]\!]^{M,w,s^*,g'} = 1$
iff $[\![\exists t_1[\mathbf{npst}(t_1) \wedge \mathrm{AT}(t_1, \mathit{arrive}'(k)) \wedge t < t_1]]\!]^{M,w,i,g'} = 1$ (where $i = [\![t]\!]^{M,w,s^*,g'} = g'(t)$)
iff there is some g'' such that $g'' = g'$ (except possibly that $g''(t_1) \neq g'(t_1)$) and

a. $[\![\mathbf{npst}(t_1)]\!]^{M,w,i,g''} = 1$ and

b. $[\![\mathrm{AT}(t, \mathit{arrive}'(k))]\!]^{M,w,i,g''} = 1$ and

c. $[\![t]\!]^{M,w,i,g''} = [\![t_1]\!]^{M,w,i,g''}$

(47) a. $[\![\mathbf{npst}(t_1)]\!]^{M,w,i,g''} = 1$ iff $i \leq [\![t_1]\!]^{M,w,i,g''}$ iff $g'(t) \leq g''(t_1)$
(since $i = g'(t)$ and $[\![t_1]\!]^{M,w,i,g''} = g''(t_1)$)

b. $[\![\mathrm{AT}(t, \mathit{arrive}'(k))]\!]^{M,w,i,g''} = 1$ iff $[\![\mathit{arrive}'(k)]\!]^{M,w,i',g''} = 1$ iff $[\![\mathit{arrive}'(k)]\!]^{M,w,i'} = 1$
(where $i' = [\![t_1]\!]^{M,w,i,g''} = g''(t_1)$)

c. $[\![t]\!]^{M,w,i,g''} < [\![t_1]\!]^{M,w,i,g''}$ iff $g'(t) < g''(t_1)$
(since $[\![t]\!]^{M,w,i,g''} = g''(t) = g'(t)$ and $[\![t_1]\!]^{M,w,i,g''} = g''(t_1)$)

Thus,

(48) (46a–c) iff

a. $g'(t) < g''(t_1)$ (from (46a,c) = (47a,c)) and

b. $[\![\mathit{arrive}'(k)]\!]^{M,w,i'} = 1$ (where $i' = g''(t_1)$) (from (46b) = (47b))

Thus,

(49) (46) iff there is some g'' such that $g'' = g'$ (except possibly that $g''(t_1) \neq g'(t_1)$) and

a. $g'(t) < g''(t_1)$ and

b. $[\![\mathit{arrive}'(k)]\!]^{M,w,i'} = 1$ (where $i' = g''(t_1)$)

In other words, (46) iff there is some time i' such that:

a. $g'(t) \leq i'$ and

b. $[\![\mathit{arrive}'(k)]\!]^{M,w,i'} = 1$

Thus,

(50) $[\![(44b)]\!]^{M,w,s^*,g} = 1$ iff there is some g' such that $g' = g$ (except possibly that $g'(t) \neq g(t)$) and

a. $g'(t) < s^*$ and

b. $[\![\mathit{leave}'(a)]\!]^{M,w,i,g'} = 1$ (where $i = g'(t)$) and

c. (46)

In other words, $[\![(44b)]\!]^{M,w,s^*,g} = 1$ iff there are some times i ($= g'(t)$) and i' ($= g''(t_1)$) such that $i < s^*$ and $i \leq i'$ and

a. $[\![\mathit{leave}'(a)]\!]^{M,w,i} = 1$ and

b. $[\![\mathit{arrive}'(k)]\!]^{M,w,i'} = 1$