(1) **Kinds of Reasons for a New Framework**

- Foundational Reasons
- Sociological Reasons
- Pedagogical Reasons
- Analytic Reasons

(2) **Two Seemingly Different Approaches**

The 20th century saw the emergence of two main kinds of theoretical approaches to grammar (cf. Pullum and Scholz):

GE: Generative-Enumerative  
aka derivational, procedural, or proof-theoretic

MT: Model-Theoretic  
aka constraint-based, declarative, or structural

(3) **In GE Frameworks:**

Well-formed linguistic entities (strings or trees) derived procedurally/algorithmically, by e.g.

- string rewriting (traditional CF-PSG)
- destructive operations on trees (all forms of transformational grammar)
- proofs with word/category pairs as axioms and string/category-pairs as theorems (most forms of categorial grammar)

(4) **In MT Frameworks:**

Well-formed linguistic entities (e.g. trees, feature structures, c-structure/f-structure pairs) are structures or models that satisfy the constraints imposed by the grammar, e.g.

- HPSG’s Head Feature Principle, Valence Principle, Nonlocal Principle, etc.
- LFG’s Completeness, Coherence, and Uniqueness criteria; and functional equations
(5) **Some GE Frameworks**

- Principles and Parameters (P&P), especially Minimalist Program (MP)
- Generalized Phrase Structure Grammar (GPSG)
- Categorial Grammar (CG), including
  - Combinatory Categorial Grammar (CCG)
  - Type Logical Grammar (TLG)
  - Pregroup Grammar

(6) **Some MT Frameworks**

- Arc Pair Grammar (APG)
- Head-Driven Phrase Structure Grammar (HPSG)
- Lexical Functional Grammar (LFG)

(7) **Extended Standard Theory/Government-Binding: A Hybrid Framework**

EST/GB theory had both GE (“generate”) and MT (“filter”) aspects.

Some GE Aspects of EST/GB:
- context-free base rules (X-theory, lexical insertion)
- move-α (A-movement, A-movement, head movement)
- conditions on movement (ECP, Subjacency, A-over-A, etc.)

Some MT Aspects of EST/GB:
- Case Filter
- θ-criterion
- Binding conditions A, B, and C

(8) **Some Foundational Questions**

- Why is grammatical theory divided this way?
- What is the connection between the two approaches?
- Is there a precise way to make this connection explicit?

The framework we propose does just that, thus revealing the GE and MT approaches to be “flip sides of the same coin.”
(9) **Factionalism in Linguistic Theory**
Besides the GE vs. MT split, there is a cross-cutting division of linguistic theory into

- “mainstream”, i.e. the current avatar of P&P, vs.
- all the other (“non-mainstream”) frameworks

(10) **Factionalism in Linguistic Theory (Continued)**
The non-mainstream frameworks do not form a countervailing coalition, but instead tend to split into ever-more-numerous and ever-smaller subgroups, e.g.

- CCG vs. TLG vs. pregroup grammar
- OT-style LFG vs. linear-logic style LFG
- RSRL-based HPSG vs. constructional HPSG, etc.

(11) **Mainstream Generative Grammar**
- is broadly influential in empirical/descriptive linguistics
- is widely credited asking questions that lead to the discovery of interesting phenomena
- tends to focuses on Big Questions, not so much on getting the details right
- relies on the linguist’s intuition to interpret the terminology and notation
- is not usually formalized (except by theoreticians who are mostly ignored by descriptive/empirical linguists)

(12) **The Informality of Mainstream Generative Grammar**
- has the advantage of broad accessibility to people with little background in math, logic, computer science
- but also has some disadvantages:
  - hard to apply to computational linguistics
  - not always clear whether it makes sense or what exactly is predicted
  - practitioners may lack the technical background to appreciate the contributions of the non-mainstream frameworks
(13) **Non-Mainstream Frameworks**

- broadly influential in computational linguistics
- practitioners tend to be put off by the informality/imprecision of mainstream generative and ‘only read it for the data’
- have a reputation for focusing on getting the details right, at the expense of addressing the Big Questions
- tend to be more comfortable with formal technicalia (math, logic, computer science).

(14) **The Formality of Non-Mainstream Frameworks**

- has some disadvantages:
  - the formality often makes the framework inaccessible to practicing empirical/descriptive linguists
  - in-house/home-brew technology insulates frameworks from each other
- but also has some advantages:
  - confidence that analyses make sense
  - determinate predictions
  - easier to adapt to the demands of computational linguistics

(15) **A Middle Ground**

The framework we will propose aims at the middle ground:

- full formal precision for those who want it
- terminology and notation that appeal strongly to the intuition of linguist who are not interested in technicalia
- formal basis in mainstream, accessible math and logic: set theory, propositional and predicate logic, and lambda calculus
- avoidance of idiosyncratic, in-house, home-brew, or otherwise nonstandard technology.
What Syntax Should be taught?

- The syntax students are taught should bridge easily to widely used frameworks (CG, LFG, MP, PSG)
- What syntax students are taught should also mesh well with mainstream semantics.
- It is not asking too much of someone who wants to be a linguist to learn a little basic, standard math (after all, it is science)
- The framework we will propose is consciously designed with these considerations in mind.

Analytic Reasons for a New Framework

- Most of the current frameworks have valuable insights and analyses to offer.
- It is not always easy to port an insight or analysis from one framework to another. e.g.
  - the CG analysis of coordination to HPSG or P&P
  - the GB or HPSG ‘binding theory’ to CG
- We propose a framework that permits both GE and MT styles of analysis, rather than forcing a choice.

Summary of Desiderata

- Basis in completely standard, accessible math and logic (basic set theory, propositional and predicate logic, lambda calculus)
  - little beyond Dowty, Wall, and Peters
- Can host analyses from wide variety of frameworks
- Neutral between GE and MT approaches
- Easily related to widely accepted linguistic concepts
- Simple, easily graspable notation that makes things intuitively clear without getting mired in technical details
- straightforward interface to mainstream semantics
(19) **The Saussurean Sign (c. 1907-1911)**

- A natural language is made up of **signs**.
- A sign is a mentally represented entity shared by members of a speech community.
- A sign consists of an associative bond between
  - a sound image or **signifiant** (mental representation of linguistic sound), and
  - a concept or **signifié**.

(20) **The Saussurean Syntagm**

- For Saussure, signs were all lexical.
- For linear combinations of two or more linguistic units, he used the term **syntagm**.
- Syntagms include:
  - word-internal combinations of morphemes
  - multi-word combinations (**phrases**).

(21) **Nobody’s Perfect**

Saussure seems not to have taken the intellectual leaps required to

- view phrases as more complex signs with **signifiants** and **signifiés** of their own, or
- to classify signs and syntagms into categories on the basis of their combinatorial potential (ability to become constituents of larger syntagms).

(22) **Linguistic Forms (Bloomfield 1933)**

- Bloomfield defined a (linguistic) **form** to be “a fixed combination of phonemes” with “a constant and definite meaning”
- This notion of form subsumes:
  - Saussure’s signs (syntactic words),
  - morphemes (forms without meaningful subparts)
  - Saussure’s syntagms, both the super-lexical (phrases) and the sublexical (multimorphemic parts of words)
(23) Constructions (Bloomfield 1933)

- “When two (or rarely more) forms are spoken together, as constituents of a complex form, the grammatical features by which they combine make up a construction.”
- “Syntactic constructions . . . are constructions in which none of the immediate constituents is a bound form.”
- “All the forms which can fill a given position [in a construction] thereby constitute a form class.”

(24) Forms and Constructions Clarified (Hockett 1958)

- “A class of forms which have similar privileges of occurrence in building larger forms is a form-class.”
- “A construction is . . . a pattern for building composite forms of a specific form-class out of ICs [immediate constituents] of specific form classes.”
- “The description of a specific construction asserts that ‘any member of such-and-such a form-class, conjoined to any member of a certain other form class, produces a form which belongs to a certain third form-class’.”

(25) Hockett 1958 (Continued)

- “All constitutes built by a single construction are necessarily members of the same form class. The form-class, however, may also include forms built by some other construction, and even single morphemes.”
- “The common feature of meaning of . . . composite forms built by a single construction is the meaning of the construction.”

(26) CF-PSGs (Chomsky 1957)

```
Sentence → NP + VP
NP → T + N
VP → Verb + NP
T → the
N → man, ball, etc.
Verb → hit, took, etc.
```
N.B.: Chomsky thought of such rules as *instructions for rewriting strings* of nonterminal (e.g. NP) and terminal (e.g. hit) symbols.

(27) Derivations
In an obvious way, CF-PSGs license *derivations* like

\[
\begin{array}{c}
\text{Sentence} \\
\text{NP} & \text{VP} \\
T & N & \text{Verb} & N \\
\text{the} & \text{man} & \text{hit} & \text{the} & \text{ball} \\
\end{array}
\]

(28) CF-PSGs from a Structuralist Perspective
Intuitively, Chomsky’s rules also lend themselves to a “structuralist” interpretation, e.g.

- The word *man* is a member of the form class N
- There is a construction that takes a (member of the form class) NP and a VP and constructs a Sentence.

In HOG, we’ll use logic to make this idea precise.

(29) Grammar as Logic (Lambek 1958)
Lambek was the first to formulate NL grammars as (certain kinds of) propositional logics —Lambek calculi (LCs).

A more descriptive name for LC is *noncommutative linear implicative intuitionistic propositional logic*.

(30) Lambek Calculus (LC)
- LC formulas for an illustrative simple LC of English:
  - NP, N, and S are atomic formulas.
  - if \( A \) and \( B \) are formulas, so are \( A/B \) and \( B\backslash A \).
  - / and \( \backslash \) are ‘directed’ analogs of the *implication* (\( \supset \)) connective of familiar propositional logic.
- LC has ‘directed’ (left and right) analogs of modus ponens and hypothetical proof (conditionalization).
• The instances of modus ponens can be expressed informally (in a way suggestive to linguists) as follows:

\[ B \rightarrow B/A \ A \]
\[ B \rightarrow A \ A\backslash B \]

(31) **Lambek Calculi as Grammars**

• Similarly to Chomsky’s CF-PSGs, Lambek used his calculi to assign nonterminals (LC formulas) to word strings.

• Words are “assigned” to one or more formulas, e.g.

\[ \text{NP}/N \rightarrow \text{the} \]
\[ N \rightarrow \text{man, ball} \]
\[ (\text{NP}/S)/\text{NP} \rightarrow \text{hit} \]

• The rules are used to assign formulas to longer word strings (example follows).

(32) **Lambek Calculus Derivation**

The following is the LC counterpart of the Chomsky (CF-PSG) derivation in (27):

Chomsky derivation for comparison:
(33) **Tectogrammar vs. Phenogrammar (Curry 1961)**

- Both CF-PSG and Lambek Calculus encode in the grammar the relative order of immediate constituents (and of the terminal yield).
- Curry (1961) proposed a kind of grammar similar to Lambek’s, but which only concerned “pure combinatorics” of elements (what he called **tectogrammar**), not the superficial form of words or the order in which they occur (**phenogrammar**).
- Tectogrammar and phenogrammar are often referred to as abstract syntax and concrete syntax.

(34) **Tectogrammar vs. Phenogrammar (continued)**

- Versions of this distinction were carried over into some forms of CG (Dowty) and HPSG (Reape, Kathol).
- A closely related idea is the factoring of phrase structure rules in GPSG into immediate dominance (ID) rules and linear precedence (LP) constraints.
- Likewise related is the GB treatment of (universal) $\bar{X}$-schemata as unordered, with order constraints as language-specific parameter settings.

(35) **Tectogrammar vs. Phenogrammar (continued)**

- The tecto vs. pheno distinction is explicitly adopted in such current frameworks such as Abstract Categorial Grammar (ACG), Lambda Grammar, Grammatical Framework (GF), and the Higher Order Grammar (HOG) framework proposed here.
- But most forms of CG (CCG, TLG, Pregroup Grammar) do not recognize the existence of tectogrammatical entities as such; instead they assign ordered pairs of a string and a meaning (or a lambda term) to syntactic categories.
(36) **Mappings from Tectogrammar**

- On Curry’s conception, there are interpretive mappings from
  - tectogrammar to phenogrammar
  - tectogrammar to semantics
- We can think of these mappings as, respectively
  - the **syntax-phonology interface** (with ‘phonology’ broadly construed to include word order)
  - the **syntax-semantics interface**

But Curry said little about these.

(37) **Chomsky Meets Curry?**

It’s tempting to view GB along Curry-esque lines by:
- identifying everything up through SS with tectogrammar
- identifying the derivation from SS to PF with the mapping from tectogrammar to phenogrammar
- identifying the derivation from SS to LF with the mapping from tectogrammar to semantics.

(38) **Chomsky Meets Curry (Continued)**

But the analogy is hard to pursue, because in GB, the **whole** SS must be derived before the derivations of PF and LF begin.

In Curry’s architecture, the more straightforward move is to compute the phonology and the semantics **compositionally**, at each step of the syntactic derivation.

(39) **Montague Meets Curry**

It is even more tempting to view **Montague Grammar** along Curry-esque lines by:
- identifying analysis trees with tectogrammar
- identifying strings with phenogrammar
- identifying IL terms with semantics.

The analogy **seems** hard to extend, because Montague defined ‘translation’ as a **relation** between strings and IL terms.
(40) Montague meets Curry (Continued)

But Montague mentioned in passing that he could just as well have defined translation as a function from analysis trees to IL terms. If this idea is carried out, then:

- Montague’s syntactic categories can be thought of as formulas in a propositional logic with two implications (/ and //).
- Analysis trees can be thought of as tectogrammar.
- Translation is the mapping from tectogrammar to semantics.
- The relationship between analysis trees and strings can also easily be formulated as a function.

To a large extent, the framework we will propose (HOG) can be seen as as revising Montague Grammar along such Curry-esque lines.

(41) Beginnings of Semantics (Frege 1892)

Some key Fregean ideas we will adopt:

- A linguistic expression expresses a sense (Sinn) (roughly, literal meaning) which is independent of contingent fact.
- The sense of an expression together with contingent fact jointly determine the reference (Bedeutung) of the expression.
- A declarative sentence’s sense is the proposition that it expresses, and its reference is the truth value of that proposition.

(42) Compositionality

- A Fregean idea we will modify:
  The reference of a phrase is a function of the references of its parts and how they are put together.
- Instead, the version of compositionality we will use is:
  The sense of a phrase is a function of the senses of its parts and how they are put together.
- This is in line with Hockett’s idea of constructional meaning.
- It is also in line with the intuition that we can figure out what an expression means without knowing how things are (i.e., independently of contingent fact).
Typed Lambda Calculi

Typed lambda calculi (TLC) are another kind of rule system, developed by Church and Curry (1930s). TLCs have:

- **Types**, which can be thought of as analogous to the nonterminals of CFG.
- **Symbols** (variables, constants, parentheses, and λ), which can be thought of as analogous to terminals.
- **Rules** for assigning types to certain strings of symbols called terms. The rules can be thought of as analogous to the phrase structure rules (PSRs) of a CFG.

Implicative TLCs

- There are different kinds of TLCs, which (as we'll see) correspond to different kinds of propositional logic (PL).
- We'll start with the simplest kind of TLC, which we'll call **implicative** TLC—the name will be explained soon.
- Implicative TLC is familiar to linguists because it underlies the higher-order logics used in formal semantics, Montague's (1970) IL and Gallin's (1975) Ty2.

Types of Implicative TLC

a. There are some **basic types** X, Y, ...

b. if A and B are types, so is $A \supset B$.

[N.B. Montague wrote $A \supset B$ as $\langle A, B \rangle$.

Term Formation Rules of Implicative TLC

We write ‘$a : A$’ for ‘the string $a$ is a term of type $A$’.

a. For each type $A$, there are **variables** $x_0^A, x_1^A, \ldots : A$

b. for each type, there can be **constants** of that type

c. Application:

   if $f : A \supset B$ and $a : A$, then $\text{app}(f, a) : B$.

d. Abstraction:

   if $x : A$ is a variable and $b : B$, then $\lambda x \, b : A \supset B$.

Note: ‘$\text{app}(f, a)$’ is usually abbreviated ‘$f(a) : B$’.
(47) Implicative TLC as Proof Theory (Curry 1934)

- The types are formulas of a propositional logic, namely implicative intuitionistic propositional logic (IIPL). (Hence the name ‘implicative TLC’.)
- The basic types are the atomic formulas.
- ⊃ is implication.
- The terms of type $A$ are proofs of the formula $A$.
- The constants are nonlogical axioms, or more precisely, proofs of them.
- Free variables in terms are undischarged hypotheses of proofs.

(48) Implicative TLC as Proof Theory (Continued)

- Application is modus ponens (implication elimination)
- Abstraction is hypothetical proof (implication introduction).
- For any type $A$, $A$ is a theorem (of IIPL) iff there is a term of type $A$ with no constants and no free variables.
- Such a term is called a combinator, and this style of proof theory is sometimes called combinatorial.

(49) Implicative TLC Thought of Structurally (Henkin 1947)

In a set-theoretic model $I$ of an implicative TLC:

- Basic types denote certain designated sets.
- $I(A ⊃ B)$ denotes some subset of the set of functions from $I(A)$ to $I(B)$.
- Terms of type $A$ are interpreted as members of $I(A)$.
- $I(f(a))$ is the result of applying $I(f)$ to $I(a)$.
- If $x : A$ and $b : B$ then $I(λx b)$ is the ‘obvious’ function from $I(A)$ to $I(B)$.
Different Kinds of TLCs for Different Logics

- So implicative TLCs provide a proof theory for IIPL.
- But in recent decades it was established (Lambek 1980, Howard 1980, Jay 1989, Wansing 1992, Gabbay and de Queiroz 1992) that a wide range of propositional logics, both weaker than IIPL (e.g., relevant logic, linear logic, Lambek Calculus) and stronger than IIPL (with more connectives and/or more rules) have their own TLCs.
- The kind of TLC appropriate to a certain kind of logic is called its Curry-Howard (proof term) calculus.

Positive TLC Overview

- Positive TLC is the Curry-Howard calculus for positive intuitionistic PL (PIPL).
- PIPL is IIPL with the addition of
  a. a logical truth formula (or unit type) T, and
  b. a conjunction connective (or type constructor) ∧.
- Positive TLC is especially suitable as a notation for linguistic meanings.
- Positive TLC also underlies the kind of higher-order logic (HOL) we will write our grammars in.

Positive TLC Types

- Each positive TLC has a finite set of basic types;
- T is a type, called the unit type;
- if A and B are types, so is the conjunctive type A ∧ B
- if A and B are types, so is the implicative type A ⊃ B.

Positive TLC Variables and Constants

Each positive TLC has:

- for each type A, variables x₀^A, x₁^A, ... of type A;
- for each type A, a designated finite set of nonlogical constants of type A and
- *, the logical constant, of type T.
(54) **Positive TLC Term Constructors**

- **Pairing:** pair\(_{A,B}\) of arity \(A, B\); \(A \land B\)
- **Left Projection:** \(\pi_{A,B}\) of arity \(A \land B; A\)
- **Right Projection:** \(\pi'_{A,B}\) of arity \(A \land B; B\)
- **Application:** app\(_{A,B}\) of arity \(A \supset B, A; B\)

The subscripts are usually omitted.

‘pair\((a, b)\)’ is usually abbreviated to ‘\((a, b)\)’.

‘app\((f, a)\)’ is usually abbreviated to ‘\(f(a)\)’.

(55) **Proof-Theoretic (Curry-Howard) Interpretation of Positive TLC**

Just as for implicative TLC, plus:

- \(\ast\) : \(T\) is the (proof of the) logical axiom of **Truth**;
- **pairing** is **Conjunction Introduction**; and
- the projections are the **Conjunction Eliminations**.

(56) **Model-Theoretic (Henkin/Montague) Interpretation of Positive TLC**

Just as for implicative TLC, plus:

- \(I(T)\) is a singleton set;
- \(I(A \land B) = I(A) \times I(B)\)

(57) **A Less Familiar Curry-Howard Calculus**

**Bilinear TLC** is the Curry-Howard calculus for Lambek Calculus (LC). The proof terms are as follows:

- variables of each type
- optional constants of each type (intuitively: words)
- there are both left and right application, so if \(f : A/B, g : B\backslash A, a : B\) then \((^L f a) : B\) and \((f a^R) : B\).
- there are both left and right lambda-abstraction, so if \(x : A\) and \(b : B\), then \(\lambda x^L b : A \backslash B\) and \(\lambda x^R b : B/A\).
- a lambda must bind exactly one variable occurrence

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(58) **The Sociology of the Bilinear TLC**

- Most categorial grammarians only use the types of Lambek Calculus, not the Curry-Howard proof terms.
- At least in part, this is because other styles of proof theory (either Gentzen sequents or proof nets) are better adapted to proving metatheorems.
- Another reason might be that, once one adopts a Curry-Howard calculus, then one wonders: what is the significance of a set-theoretic interpretation of the calculus? The types are interpreted as sets, but sets of what?

(59) **Curry Meets Bloomfield**

- For example, suppose a categorial grammar tells us:
  
  \[ \text{Fido} : \text{NP}, \text{barks} : \text{NP}\backslash\text{S} \]

- Then \( (L \text{ Fido barks}) : \text{S} \), and therefore

\[ I(L \text{ Fido barks}) \in I(S). \]

- But, in linguistic terms, what exactly is \( I(L \text{ Fido barks}) \)?

  - First answer: \( I(S) \) is a Bloomfield/Hockett form class, and \( I(L \text{ Fido barks}) \) is one of the forms in it.
  - Second answer: \( I(L \text{ Fido barks}) \) is an abstract syntactic entity, i.e. a Curry tectogrammatical structure.

- We take these to be saying the same thing.

(60) **TLCs for Syntax**

- Using Curry-Howard terms to denote abstract syntactic entities is inconsistent with mainstream CG (CCG, TLG, Pregroup Grammar).
- That’s because most categorial grammarians do not believe in abstract syntactic entities.
- Instead, they mostly think of grammars as generating string/type pairs (or string/type/meaning triples).
- But many 21st-century frameworks do use the Curry-Howard terms of their respective syntactic logics.
TLCs for Syntax (continued)

Frameworks that use the Curry-Howard proof terms of their syntactic logics include:

- Grammatical Framework (GF, Ranta et al.)
- Abstract Categorial Grammar (ACG, De Groote et al.)
- Lambda Grammar (Muskens)
- Minimalist Categorial Grammars (Lecomte & Retoré)
- Higher Order Grammar (HOG)

In set-theoretic interpretations, the types denote syntactic categories (form classes) and the terms denote the abstract syntactic entities (forms) that belong to them.

What is Different about HOG

What distinguishes HOG from the other frameworks that make use of a syntactic Curry-Howard calculus is this:

In HOG, the TLC is embedded inside a higher order logic (HOL).

This HOL can be used:

a. to define subtypes of linguistic entities
b. to axiomatize the compositionality of the syntax-phonology and syntax-semantics interfaces.

Approaching HOG

We'll work up to HOG in stages. At the first stage, we try to convey something of the flavor of the framework by taking a simple CF-PSG and showing how it can naturally be thought of in a structural (or declarative, or model theoretic) way.
(64) A Traditional CF-PSG

\[ T = \{ \text{Fido, Felix, Mary, barked, bit, believed, the, cat, dog} \} \]
\[ N = \{ S, \text{NP, VP, TV, DTV, SV, Det, N} \} \]

Lexicon:

- NP \rightarrow \{ \text{Fido, Felix, Mary} \}
- VP \rightarrow \text{barked}
- TV \rightarrow \text{bit}
- DTV \rightarrow \text{gave}
- SV \rightarrow \text{believed}
- Det \rightarrow \text{the}
- N \rightarrow \{ \text{cat, dog} \}

Rules:

- S \rightarrow \text{NP VP}
- VP \rightarrow \text{TV NP}
- VP \rightarrow \text{DTV NP NP}
- VP \rightarrow \text{SV S}
- NP \rightarrow \text{Det N}

(65) A Structuralist Reconceptualization

- Nonterminals are names of form classes.
- Lexical entries are declarations that a certain word (lexical form) belongs to a certain form class.
- CG-PSRs are functions (constructions) that map certain cartesian products of form classes to other form classes.
- The rules are purely tectogrammatical: they only tell what forms can combine to make other forms.
- How forms sound and what they mean will have to be specified elsewhere in the grammar (in the interfaces to phonology and semantics, respectively).
(66) **A Structuralist CF-PSG**

Types (names of form classes):
{S, NP, VP, TV, DTV, SV, Det, N}

Constants (names of words):
- Fido, Felix, Mary : NP
- barked : VP
- bit : TV
- gave : DTV
- believed : SV
- the : Det
- cat, dog : N

(67) **A Structuralist CF-PSG (Continued)**

Rules (names of constructions)
- p : NP, VP; S
- q : TV, NP; VP
- r : DTV, NP, NP; VP
- s : SV, S; VP
- t : Det, N; NP

(68) **A structuralist CF-PSG derivation**

\[ p(Mary, s(believed, p(Fido, q(bit, t(the, cat)))))) : S \]

(69) **A Traditional CF-PSG Derivation (for Comparison)**

\[
\begin{aligned}
S & \quad \text{NP} & \quad \text{VP} \\
 & \quad \text{Mary} & \quad \text{SV} & \quad \text{S} \\
 & \quad \text{believed} & \quad \text{NP} & \quad \text{VP} \\
 & \quad Fido & \quad \text{TV} & \quad \text{NP} \\
 & \quad \text{bit} & \quad \text{Det} & \quad \text{N} \\
 & \quad \text{the} & \quad \text{cat} & \quad \text{N} \\
\end{aligned}
\]
(70) **HCG Overview**

- Like TLC, HCG has both **types** and **terms**.
- Proof-theoretically:
  - types are formulas
  - terms are proofs.
- Model-theoretically:
  - types denote form syntactic categories (form classes)
  - terms denote syntactic forms (tectogrammatical words and phrases).
- But HCG isn’t quite a TLC, because it doesn’t have lambda abstraction (that will come later).

(71) **HCG Overview (continued)**

- As in CG, there are multiple implications.
- But they do not correspond to directionality. Rather, they correspond to different **grammatical relations**, or, in HPSG terms, different **valence features**, e.g. \( \text{\textbackslash subj} \) (subject), /\text{\textcomp} \) (complement), etc.
- There are multiple conjunctions, for different ways of ‘clumping’ forms together (e.g. one way for ‘sister’ complements, another for coordinated conjuncts).
- The precise inventories of basic terms, basic types, implications, and conjunctions in a HCG depend on the language and the fragment being analyzed.

(72) **HCG Types**

- NP, S, and N are basic types.
- If \( A \) and \( B \) are types, so are \( A\text{\textbackslash subj} B, A/\text{\textcomp} B, \) and \( A/\text{\textspec} B \). These denote categories of forms that seek a valent of a particular grammatical relation to combine with,
- If \( A \) and \( B \) are types, so is \( A \circ B \). This corresponds to an HPSG list of length two (e.g. two sister complements).
- For lists of length \( > 2 \), we adopt the convention that \( A_0 \circ \ldots \circ A_{n+1} \) means \( (A_0 \circ \ldots \circ A_n) \circ A_{n+1} \).
(73) HCG Constants (Lexicon)

Fido, Felix, Mary : NP
barked : NP/subj S
bit : (NP/subj S)/comp NP
gave : (NP/subj S)/comp (NP ∘ NP)
believed : (NP/subj S)/comp S
the : NP/spec N
cat, dog : N

(74) HCG Term Constructors (Rules)

- ·_{A,B} of arity A, B; A ∘ B;
- app^{subj}_{A,B} of arity A, A/subj B; B;
- app^{spec}_{A,B} of arity B/spec A, A; B and
- app^{comp}_{A,B} of arity B/comp A, A; B.

(75) HCG Term-Constructor Abbreviations

- The parenthesis convention for · is like the one for ∘:
  \(a_0 \cdot \ldots \cdot a_{n+1}\) means \((a_0 \cdot \ldots \cdot a_n) \cdot a_{n+1}\).
- app^{subj}(a, f), app^{spec}(f, a), app^{comp}(f, a) are written
  \((\text{subj} a f), (f a \text{spec}), (f a \text{comp})\) respectively.

(76) Terms of HCG vs. CF-PSG Labelled Bracketings

CF-PSG labelled bracketing:
\([_S \text{Mary} [_{VP} \text{believed} [_{S \text{Fido} [_{VP} \text{bit} [_{NP} \text{the cat}]]]]]]]

HCG term:
\((\text{subj} \text{Mary} (\text{believed} (\text{subj} \text{Fido} (\text{bit} (\text{the cat} \text{spec} \text{comp})) \text{comp})))\)
(77) Arboreal Equivalent for HCG Term

```
S
  └── NP
    ├── NP\_SUBI\_S
    │   ├── (NP\_SUBI\_S)/COMP\_S
    │       └── S
    │          ├── believed
    │          │   └── NP
    │          │       └── NP\_SUBI\_S
    │          │          ├── Fido
    │          │          │   └── (NP\_SUBI\_S)/COMP\_NP
    │          │          │       └── bit
    │          │          │           └── NP\_SPEC\_N
    │          │          │               └── N
    │          │          └── the
    │              └── cat
    └── Mary
```

(78) A Multiple-Complementation Example

a.  
```plaintext
(a. (\SUBJ Mary (gave (Fido \_ Felix) \COMP)))
```

b.  
```plaintext
S
  └── NP
    ├── NP\_SUBI\_S
    │   ├── (NP\_SUBI\_S)/COMP\_NP\_NP\_NP
    │       └── gave
    │          └── NP\_SPEC\_N
    │              └── Fido
    └── Mary
```

c. The presence of \_ for types and \_ for terms means that the grammar writer gets to choose whether to ‘curry’ multicomplement verbs, i.e. whether they take their complements one at a time (as in most CGs) or simultaneously (as in most PSGs).

(79) GIANT RED FLAG

HCG trees are NOT the phrases! They are just an alternative notation for terms, which DENOTE the phrases in an interpretation!

Failing to make this distinction is a mistake on a par with mistaking Montague’s IL constant ‘love’ for ‘the real thing’.
HCG is Neutrally Proof-Theoretic or Model-Theoretic

Proof-theoretically:

- types are formulas of a propositional logic
- the type constructors are logical connectives (a nonassociative, noncommutative, nonidempotent conjunction and three implications)
- constants are (proofs of) nonlogical axioms
- terms (or trees) are proofs and their types are theorems
- the term constructors are logical rules (ways of building new proofs from old ones): · is Conjunction Introduction and the $(\neg \neg)$ are flavors of Modus Ponens.

Model-theoretically:

- types denote form classes (syntactic categories)
- each type constructor denotes a function a that maps each ordered pair of form classes to a form class
- constants denote lexical forms (syntactic words)
- terms (or trees) denote syntactic forms (words and phrases);
- each term constructor is a (type-indexed) function (or construction) from a cartesian product of form classes to a form class.

We Cannot Yet Assert Anything about Language

HCGs provide a neat notation for forms and form classes. But we can’t say anything about them, e.g.

a. how words sound
b. in what order the phonologies of words appear when the words combine to form other forms
c. what words mean
d. how meaning composition works
e. which categories are subsets of which other categories
f. how entailment works
g. how reference is related to semantic composition
(83) **Two Precedents**

There are at least two well-known precedents in linguistics for using a classical quantified logic to make assertions about the objects of study: Montague semantics and HPSG.

(84) **First Precedent: Montague Semantics**

- the logic is a classical higher-order predicate logic, either Montague’s IL or Gallin’s Ty2.
- There are terms whose model-theoretic interpretations are (mathematical models of) meanings.
- there are (term-level) nonlogical axioms—meaning postulates—that express entailments.
- Ty2 is accessible and well-understood, being based directly on Henkin’s (1950) reaxiomatization of Church’s (1940) simple theory of types.

(85) **Second Precedent: HPSG**

- The logic is RSRL (Richter 2000), an HPSG-specific classical logic with (bounded) quantification and an extremely weak type system (no implicative types).
- The constraints (nonlogical axioms) can be about all kinds of linguistic entities (e.g. syntactic, semantic, morphological, phonological, pragmatic).
- To represent semantic objects, Ty2 and lambda conversion can be simulated inside of RSRL.
- RSRL is little known beyond the HPSG community.
- RSRL has a highly nonstandard syntax and semantics.

(86) **The Logic We will Write Linguistic Theories in:**

- is like Ty2 plus conjunctive types and subtypes.
- The type system includes a semantic ontology similar to Montague’s and a syntactic ontology like that of HCG.
- Like RSRL, the logic is used to talk about all kinds of linguistic entities and connections between them.
• In particular (unlike Montague semantics) the theories of the syntax/phonology and syntax/semantics interfaces can be directly asserted as nonlogical axioms.

• The logic is easily understood by anyone who understands basic Montague semantics.

(87) **The Standard Way to Extend a TLC to an HOL**

This general approach goes back to Church (1940).

• Start with a TLC
• add a type Bool for (term-level) formulas
• add equality symbols for all types
• add suitable axioms about equality
• *define* the usual classical logical connectives and quantifiers in terms of \( \lambda \) and equality.

Caution: the word ‘formula’ is now ambiguous between ‘type’ (formula of the PIPL type logic) and ‘boolean term’ (formula of the classical higher-order term logic).

(88) **A Logic Defined in this Way:**

• has all the term equalities expected in TLC (‘conversion’)
• has all the theorems of classical first-order logic
• allows quantification over variables of all types.

(89) **Historical Synopsis of Classical HOL**

• in his 1947 dissertation, Church’s student Henkin
  – reaxiomatized Church’s STT, adding a key axiom (*Boolean Extensionality*) identifying boolean identity with bi-implication, and
  – proved completeness relative to the class of set-theoretic models that bear his name;

• Henkin’s student Gallin (1975) showed that Henkin’s HOL with two basic types (besides Bool) instead of just one was equivalent to Montague’s IL; and

• Groenendijk and Stokhof (1980s) started using Ty2 instead of IL for NL semantics.
Lambek and Scott 1986
Lambek and Scott extended HOL by:
  • not insisting on Excluded Middle in the term logic
  • adding a mechanism for subtypes analogous to the Axiom of Separation in set theory
  • considering a wider class of (not necessarily set-theoretic) models, viz. toposes, generalizing the Henkin models.

The HOL we will use has:
  • everything Ty2 has (but different basic types for meanings)
  • everything positive TLC has
  • everything HCG has
  • subtypes similar to those of Lambek and Scott
  • constants for words, their phonologies, and their meanings.

HOL: a Closer Look
a. Start with a positive TLC.
b. Add a type Bool (Montague’s $t$).
c. Boolean terms are called formulas.
d. In an interpretation $I$, $I(\text{Bool})$ is called the set of truth values.
e. Axioms will ensure that $I(\text{Bool})$ has exactly two members.
f. For each type $A$, add a constant $=_A$: $(A \land A) \supset \text{Bool}$. This is written infix-style ($a = b$). The type subscript is usually omitted.
g. $I(=_A)$ is the identity relation on $I(A)$.

Defining the Classical Connectives and Quantifiers
Here $\phi$ is a metavariable over boolean terms, $x$ is a variable of type $A$, and $p, q$ are variables of type Bool:
a. true $= \text{def} \star = \star$;
b. $\forall x \phi = \text{def} \lambda x \phi = \lambda x \text{true}$;
c. false $= \text{def} \forall p p$
d. $\phi \land \psi = \text{def} (\phi, \psi) = (\text{true}, \text{true})$;
e. $\phi \lor \psi = \text{def} \phi = (\phi \land \psi)$;
f. $\phi \leftrightarrow \psi = \text{def} [(\phi \lor \psi) \land (\psi \lor \phi)]$;
g. $\sim \phi = \text{def} \phi \supset \text{false}$;

h. $\phi \lor \psi = \text{def} (\sim \phi) \land (\sim \psi)$; and

i. $\exists x \phi = \text{def} \sim \forall x \sim \phi$.

(94) **Options for Axiomatizing HOL**

- Gallin (Ty2, 1975) follows Henkin (1950);
- Carpenter (1997) follows Andrews (1986);
- Lambek and Scott (1986) have conjunction in the underlying type logic, subtyping, and the option of dropping Excluded Middle.
- We’ll remain agnostic about how to axiomatize HOL.
- We’ll assume general familiarity with Ty2 and mention a few useful theorems (or axioms, depending on the choice of axiomatization).
- We write $\vdash \phi$ to mean ‘$\phi$ is provable in HOL’.

(95) **Equality is an Equivalence Relation**

a. $\vdash a = a$ (reflexivity);

b. if $\vdash a = b$, then $\vdash b = a$ (symmetry);

c. If $\vdash a = b$ and $\vdash b = c$, then $\vdash a = c$ (transitivity);

(96) **Rules for Substitution of Equals**

a. if $\vdash a = c$ and $\vdash b = d$, then $\vdash (a, b) = (c, d)$;

b. if $\vdash f = g$ and $\vdash a = b$, then $\vdash f(a) = g(b)$;

c. if $\vdash a = b$, then $\vdash \lambda x a = \lambda x b$;

(97) **Axioms for Cartesian Products**

a. $\vdash a = *$ for all $a : T$;

b. $\vdash \pi(a, b) = a$;

c. $\vdash \pi'(a, b) = b$;

d. $\vdash (\pi(c), \pi'(c)) = c$;
(98) Axioms for Lambda Conversion
   a. \((\beta) \vdash [\lambda_{x \in A}\phi[x]](a) = \phi[a]\) if \(a : A\) is substitutable for \(x\);
   b. \((\eta) \vdash \lambda_{x \in A}f(x) = f\) for all \(f : A \supset B\) provided \(x\) does not occur freely in \(f\); and
   c. \((\alpha) \vdash \lambda_{x \in A}\phi[x] = \lambda_{y \in A}\phi[y]\) if \(y\) is substitutable for \(x\).

(99) Axiom of Excluded Middle
   \(\vdash \forall p (p \lor \sim p)\)

(100) Axiom of Nondegeneracy
     \(\vdash \sim (\text{true} = \text{false})\)

(101) Axioms for Boolean Equality
   a. \(\vdash \phi = (\phi = \text{true})\);
   b. If \(\vdash \phi\) and \(\vdash \phi = \psi\), then \(\vdash \psi\);
   c. \(\vdash \phi\) iff \(\vdash \phi = \text{true}\); and
   d. \(\vdash \forall p,q [(p \leftrightarrow q) \supset (p = q)]\) (Boolean Extensionality)

(102) Motivation for Subtypes
   • Standard HOL has no way to say \(A\) is a subtype of \(B\).
   • In an interpretation \(I\), this should mean \(I(A) \subseteq I(B)\).
   • Syntactic example: we want to say that the type \(NP_{\text{acc}}\) of NPs that can be objects of verbs is a subtype of \(NP\).
   • Semantic example: we want to say that the type \(\text{World}\) is a subtype of the type \(\text{Prop} \supset \text{Bool}\) of sets of propositions (namely the ones which are maximal and consistent).

(103) Subtypes (after Lambek and Scott 1986)
If \(A\) is a type and \(a\) an \(A\)-predicate (i.e. a closed term of type \(A \supset \text{Bool}\)), then
   • \(A_a\) is a type
   • \(\text{embed}_a\) is a term of type \(A_a \supset A\); and
   • Axioms:
     \(\vdash \forall y,z \in A_a [(\text{embed}_a(y) = \text{embed}_a(z)) \supset y = z]\)
     \(\vdash \forall x \in A [a(x) \leftrightarrow \exists y \in A_a x = \text{embed}_a(y)]\)
What Subtypes Mean in an Interpretation $I$

- $I(a)$ is a function from $I(A)$ to truth values
- $I(\text{embed}_a)$ is a one-to-one function from $I(A_a)$ to $I(A)$
- the members of $I(A)$ that $I(a)$ maps to $I(\text{true})$ are the ones that are embedded images of members of $I(A_a)$.

So $I(\text{embed}_a)$ is the function that embeds into $I(A)$ the subset whose characteristic function is $I(a)$.

HOG Architecture

- A HOG for a NL is a theory expressed in HOL.
- The theory is about the NL’s syntactic forms, including how they sound, what they mean, and how they combine.
- A model of a NL is a set-theoretic model of the theory.
- There are types for different kinds of syntactic entities (forms), phonological entities, and semantic entities.
- The linguistic entities are denoted by terms.
- There are constants denoting words, their phonologies, and their meanings.
- There are term constructors (rules) for combining linguistic entities to form other ones.
- The theory of the syntax-semantics and syntax-phonology interfaces (how forms sound and what they mean) consist of:
  - (unquantified) equations about specific words; and
  - universally quantified equations about forms constructed by particular rules: these characterize the compositionality of phonology and of semantics.

The Usual Syntactician’s Simplification of Phonology

- Phonologies of syntactic forms are strings of phonological words.
- The phonology of a phrase is the concatenation (in some order) of the phonologies of the immediate constituents.
- For now we ignore stress, intonation, sandhi, and nonconcatenative forms of phonological composition.
Technically: Phonology is a Monoid

a. We have a basic type Phon for phonologies.
b. We have a constant $\epsilon : \text{Phon}$ for phonological zero.
c. We have constants for word phonologies, e.g.
$/$fajdo$/$, $/$blt$/$, $/$fillks$/$, ...$ : \text{Phon}$
d. We have a constant $\sim : (\text{Phon} \land \text{Phon}) \supset \text{Phon}$ for concatenation of phonologies, and the following axioms:
   i. $\vdash \forall x, y, z ((x \sim y) \sim z = x \sim (y \sim z))$
   ii. $\vdash \forall x (\epsilon \sim x = x)$
   iii. $\vdash \forall x (x = \epsilon \sim x)$

Defining the Mapping from Forms to Phonologies

a. Intuitively, phonological interpretation is a function from forms to phonologies.
b. But we cannot denote this function by a constant of type Form $\supset$ Phon, because there is no type Form! Instead, there are many different form types.
c. So for each form type $A$, we will have a constant $\text{phon}_A : A \supset \text{Phon}$ that denotes the phonology function for one type of form (e.g. NP).
d. In other words, phon is really a type-indexed family of constants $\{\text{phon}_A \mid A \in \text{FORM}\}$, where FORM is the set of form types.
e. But which types are form types?

The Notion of a Kind

A kind is a recursively defined set of types.

Definition of the Kind FORM

a. $S, \text{NP}, N \in \text{FORM}$;
b. if $A, B \in \text{FORM}$, then so are $A \circ B$, $A/_{\text{COMP}} B$, $A/_{\text{SPEC}} B$, and $A \setminus_{\text{SUBJ}} B$;
c. if $A \in \text{FORM}$ and $a : A \supset \text{Bool}$, then $A_a \in \text{FORM}$.

Specifying the Phonology of Words

$\vdash \text{phon}($Fido$) = /fajdo/$
$\vdash \text{phon}($bit$) = /blt/$
$\vdash \text{phon}($Felix$) = /fillks/ \text{ etc.}$
(112) Specifying the Phonology of Phrases

\[ \vdash \forall x, y \text{phon}(x \cdot y) = \text{phon}(x) \concat \text{phon}(y); \]
\[ \vdash \forall x, f \text{phon}^{\text{SUBJ}}(x, f) = \text{phon}(x) \concat \text{phon}(f); \]
\[ \vdash \forall f, x \text{phon}^{\text{COMP}}(f, x) = \text{phon}(f) \concat \text{phon}(x); \]
\[ \vdash \forall f, x \text{phon}^{\text{SPEC}}(f, x) = \text{phon}(f) \concat \text{phon}(x); \]
\[ \vdash \forall x \in A \text{phon}(x) = \text{phon}(\text{embed}_b(x)). \]

In plain English:

- form lists are realized in order
- subjects precede heads
- complements and specifics follow heads
- rules that embed a form of a certain category into a larger category do not affect phonology.

(113) Notational Simplification

a. Because of the associativity of concatenation, it makes no difference how we group phonologies:
\[ /fajdo/ \concat /blt/ \concat /fillks/ = /fajdo/ \concat (/blt/ \concat /fillks/) \]
b. Therefore we can omit the parentheses:
\[ /fajdo/ \concat /blt/ \concat /fillks/ \]
c. As a further simplification, we express this as simply:
\[ /fajdo\ blt\ fillks/. \]

(114) A Simple Exercise

Prove that:
\[ \vdash \text{phon}^{\text{SUBJ}}(\text{Fido (bit Felix \text{COMP})}) = /fajdo\ blt\ fillks/. \]

(115) Semantic Basics

a. For now, we ignore context-dependent/dynamic aspects, including:
   i. how what a form expresses depends on the context; and
   ii. how an utterance changes the context.

b. The second simplification means we disregard meaning in the broader sense of context change potential and consider only literal meaning.

c. The first simplification means we ignore the distinction between a form’s literal meaning and the sense expressed by an utterance of it when the meaning’s parameters are fixed by the context.
d. With Frege we assume that senses exist independently of language and independently of contingent fact (how things happen to be).

e. Also with Frege we assume that a sense together with a way things are jointly determine an extension.

f. We call the sense a form (utterance) expresses its interpretation, and the extension of that, its reference.

(116) Technical Assumptions

a. *Pace* Frege, we take interpretation, not reference, to be compositional.

b. That’s because you can figure out what a sentence means without knowing the contingent facts about the subject matter.

c. More precisely: the interpretation of a phrase is determined by the interpretations of the immediate constituents and the semantic contribution of the (tectogrammatical) rule that combined them.

d. Correspondingly, we postpone our account of reference until the notion of possible world has been introduced.

e. This is unproblematic because for us—unlike Montague, who modelled senses as intensions (functions from the set of possible worlds)—interpretations do not involve possible worlds at all.

f. Instead, we model form interpretations as hyperintensions, which make finer meaning distinctions than intensions.

(117) The Kind HYPER of Hyperintensions

a. Like forms, hyperintensions form a kind, called HYPER.

b. Ind, Prop ∈ HYPER;

c. T ∈ HYPER;

d. If A, B ∈ HYPER, so are A ∧ B and A ⊃ B; and

e. If A ∈ HYPER and a : A ⊃ Bool, then A_a ∈ HYPER.

(118) Semantic Interpretation

a. Like phonological interpretation, we treat semantic interpretation in terms of a type-indexed family of constants sem_A (A ∈ FORM).

b. But unlike the phon_A : A ⊃ Phon, which all have the same result type (Phon), the result type of sem_A has to depend on A, because different types of forms have different types of meanings.
So we treat semantic interpretation as a type-indexed family of constants \( \text{sem}_A : A \supset \text{Sem}(A) \), where \( \text{Sem} \) is a function from \( \text{FORM} \) to \( \text{HYPER} \).

(119) **Assignment of Meaning Types to Form Types**

a. \( \text{Sem}(	ext{It}) =_{\text{def}} \text{T} \)
b. \( \text{Sem}(	ext{There}) =_{\text{def}} \text{T} \)
c. \( \text{Sem}(	ext{NP}) =_{\text{def}} \text{Ind} \)
d. \( \text{Sem}(	ext{S}) =_{\text{def}} \text{Prop} \)
e. \( \text{Sem}(	ext{N}) =_{\text{def}} \text{Ind} \supset \text{Prop} \)
f. \( \text{Sem}(A \circ B) =_{\text{def}} \text{Sem}(A) \land \text{Sem}(B) \)
g. \( \text{Sem}(A/\text{COMP} \ B) =_{\text{def}} \text{Sem}(B) \supset \text{Sem}(A) \)
h. \( \text{Sem}(A/\text{SPEC} \ B) =_{\text{def}} \text{Sem}(B) \supset \text{Sem}(A) \)
i. \( \text{Sem}(A_{\text{SUBJ}} \ B) =_{\text{def}} \text{Sem}(A) \supset \text{Sem}(B) \)
j. \( \text{Sem}(A_a) =_{\text{def}} \text{Sem}(A) \).

(120) **Constants used in the Interpretations of Words**

- Mary’, Fido’, Felix’ : Ind
- barked’ : Ind \supset Prop
- bit’ : (Ind \land Ind) \supset Prop
- believed’ : (Ind \land Prop) \supset Prop
- gave’ : (Ind \land Ind \land Ind) \supset Prop

(121) **Specifying the Interpretations of Words**

N.B. Here \( x, y, z : \text{Ind} \) and \( p : \text{Prop} \).

\[
\vdash \text{sem}(\text{it}_{\text{pleo}}) = \ast \\
\vdash \text{sem}(\text{there}_{\text{pleo}}) = \ast \\
\vdash \text{sem}(	ext{Mary}) = \text{Mary’} \\
\vdash \text{sem}(	ext{barked}) = \text{barked’} \\
\vdash \text{sem}(	ext{bit}) = \lambda y.\lambda z.\text{bit’}(x,y) \\
\vdash \text{sem}(	ext{believed}) = \lambda p.\lambda z.\text{believed’}(x,p) \\
\vdash \text{sem}(	ext{gave}) = \lambda(y,z).\lambda v.\text{gave’}(x,y,z)
\]

The translations of complement-taking verbs have to be ‘lambda’d up’ since syntactically the verbs combine first with all the complements, then with the subject.
(122) Specifying the Interpretation of Phrases

\[ \forall x, y \left[ \text{sem}(x \cdot y) = (\text{sem}(x), \text{sem}(y)) \right] \]
\[ \forall x, f \left[ \text{sem}(\text{subj} x f) = \text{sem}(f)(\text{sem}(x)) \right] \]
\[ \forall f, x \left[ \text{sem}(\text{comp} f x) = \text{sem}(f)(\text{sem}(x)) \right] \]
\[ \forall f, x \left[ \text{sem}(\text{spec} f x) = \text{sem}(f)(\text{sem}(x)) \right] \]
\[ \forall x \in A \left[ \text{sem}(x) = \text{sem}(\text{embed}_a(x)) \right] \]

In English:

- Lists are interpreted as tuples.
- Valence satisfaction is interpreted as application
- Embedding into a supertype doesn’t affect meaning.

(123) A Simple Worked Example

\[ \text{sem}(\text{subj} \text{ Mary} (\text{gave Fido} \cdot \text{Felix} \text{ comp})) = \]
\[ [\text{sem}(\text{gave Fido} \cdot \text{Felix}^{\text{comp}})](\text{sem}(\text{Mary})) = \]
\[ [\text{sem}(\text{gave})](\text{sem}(\text{Fido}), \text{sem}(\text{Felix}))(\text{sem}(\text{Mary})) = \]
\[ [\lambda_{(y,z)} \lambda_x \text{gave'}(x, y, z)](\text{Fido}', \text{Felix'})(\text{Mary'}) = \]
\[ [\lambda_x \text{gave'}(x, \text{Fido'}, \text{Felix'})](\text{Mary'}) = \]
\[ \text{gave'}(\text{Mary'}, \text{Fido'}, \text{Felix'}) \]
Features in Syntactic Theory

- The theory of syntactic features plays a central role in “getting the details right” in at least three widely used frameworks: P&P, LFG, and HPSG.
- Usually in CG, that level of detail is glossed over with a footnote or brief comment to the effect that features could be handled ‘in an obvious way using unification’.
- In HOG there are no features as such, but the work features do in other frameworks gets done anyway.

Features in HPSG

- head features (SYNSEM|LOC|CAT|HEAD)
  E.g. CASE, VFORM
  ‘Major category’ distinctions (noun, verb, etc.) treated as subtypes of head
- valence features (SYNSEM|LOC|CAT|VAL)
  E.g. SUBJ, COMP, SPR
- nonlocal features (SYNSEM|NONLOC)
  E.g. SLASH, QUE, REL
- index features (SYNSEM|LOCAL|CONTENT|INDEX)
  E.g. PERS, NUM, GEND

Features in HOG

- ‘Head features’ are predicates of form types.
- ‘Valence features’ are implicative type constructors that have modus ponens but not hypothetical proof.
- ‘Nonlocal features’ are implicative type constructors that have hypothetical proof but not modus ponens.
- The status of index features is under investigation.

Basics of Head Features in HOG

- Recall: for any type $A$, an $A$-predicate is a closed term $a : A \supset \text{Bool}$. 

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• We define a type to be **simple** if it is a subtype of a basic type (this includes basic types as a special case).
• HOG treats head features as **predicates of simple form types**.
• Examples: What HPSG handles as values of:
  - **CASE**, are treated as NP-predicates
  - **VFORM**, are treated as S-predicates

(128) **Expanded Inventory of Basic Form Types**

- NP (does not include pleonastics)
- N (has phrasal members as in CG; there is no Ń)
- S (declarative sentences, including nonverbal ‘small clauses’)
- Q (interrogative sentences)
- åS (*that*-S)
- It, There (types for pleonastics)

(129) **The Boolean Connectives Generalized**

For any type \( A \) and \( A \)-predicates \( a \) and \( b \) (with \( x : A \)):

a. \( a \land b =_{\text{def}} \lambda x(a(x) \land b(x)) \)

b. \( a \lor b =_{\text{def}} \lambda x(a(x) \lor b(x)) \)

c. \( \lnot a =_{\text{def}} \lambda x \lnot a(x) \)

(130) **Basic Set Operations on Subtypes**

For any type \( A \) and \( A \)-predicates \( a \) and \( b \):

a. \( A_a \cap A_b =_{\text{def}} A_a \land b \)

b. \( A_a \cup A_b =_{\text{def}} A_a \lor b \)

c. \( A_a \setminus A_b =_{\text{def}} A_a \land (\lnot b) \)

N.B. These operations are **local**, in the sense of being defined only for two subtypes of a given type.

(131) **Where Can NPs Occur?**

First Approximation (assuming the only verbs are past-tense main verbs): We cross-classify NPs according to whether or not they can occur in the following positions (for now ignoring reflexive, relative, and interrogative pronouns):

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• subject of finite verb ("nominative")
• object of verbs or prepositions ("accusative")
• the ‘possessee’ of -’s (let’s call these “essable”)

(132) **Subtypes of NP**

Based on the distributional criteria in (131), we can identify the following five maximally specific subtypes of NP:

a. NP$_{pnom}$: purely nominative, i.e. nominative but neither accusative nor essable. Examples: *I, he, she, we, they*.

b. NP$_{neut}$: case-neutral, i.e. both nominative and accusative but not essable. Examples: *you, it, mine, yours, his$_1$, hers, ours, theirs*, as well as all NPs formed by adding -’s to an essable NP.

c. NP$_{pacc}$: purely accusative, i.e. accusative but neither nominative nor essable. Examples: *me, him, her$_1$, us, them*.

d. NP$_{npro}$: nonpronominal, i.e. nominative, accusative, and essable. Examples: *Mary, the dog, the dogs, dogs* (qua NP), *water* (qua NP).

e. NP$_{recp}$: the reciprocal pronoun *each other*, accusative and essable but not nominative.

(133) **The Familiar English Cases are Definable**

a. nom =$_{def}$ pnon $\lor$ neut $\lor$ npro

b. acc =$_{def}$ pacc $\lor$ neut $\lor$ npro $\lor$ recp

c. ess =$_{def}$ npro $\lor$ recp

---

$^1$The NP as in *I saw his*, not the determiner *his$_2$*.

$^2$The NP as in *I saw her*, not the determiner *her$_2$*. 

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(134) **Equivalently, Make the Familiar Cases Constants**
and define the (predicates corresponding to the) maximally specific subtypes:

a. $p_{nom} = \text{def} \ nom \land (\sim acc) \land (\sim ess)$
b. $p_{acc} = \text{def} \ acc \land (\sim nom) \land (\sim ess)$
c. $n_{pro} = \text{def} \ nom \land acc \land ess$
d. $n_{eut} = \text{def} \ nom \land acc \land (\sim ess)$
e. $r_{ecp} = \text{def} \ acc \land ess \land (\sim acc)$

(135) **Some Observations about English Case**

a. Cases are NP-predicates, each of which denotes (the characteristic function of) the set of noun phrases that can occur in a certain ‘syntactic position’.

b. There is no ‘case feature’ in the sense of a function that maps each NP to an abstract ‘value’, or a path leading to a ‘feature structure node’ inside the NP.

c. Morphosyntactic syncretism explained: a case-neutral NP (for English, all nonpronoun NPs) is not ambiguous between an $NP_{nom}$ and an $NP_{acc}$; rather, it is both at once (type $NP_{nom} \cap NP_{acc}$).

(136) **An Illustrative HOG Lexicon**

| i, he, she, . . . | $NP_{pnom}$ |
| you, it, mine, . . . | $NP_{neut}$ |
| me, him, her, . . . | $NP_{pacc}$ |
| Fido, Felix, Mary, · · · | $NP_{npro}$ |
| eachother | $NP_{rrecp}$ |
| barked | $NP_{nom} \setminus \text{SUBJ } S_{fin}$ |
| bit | $(NP_{nom} \setminus \text{SUBJ } S_{fin})/\text{COMP } NP_{acc}$ |
| gave | $(NP_{nom} \setminus \text{SUBJ } S_{fin})/\text{COMP } (NP_{acc} \circ NP_{acc})$ |
| believed | $(NP_{nom} \setminus \text{SUBJ } S_{fin})/\text{COMP } S_{fin}$ |
| the | $NP_{npro}/\text{SPEC } N$ |
| cat, dog | $N$ |
(137) **Properties of Possessive -’s**

a. It combines with any any essable NP (NP<sub>ess</sub>).

b. The result of the combination can occur wherever a possessive pronoun (like *mine*) or a possessive determiner (like *my*) can.

c. Phonologically, it forms a unit with the last phonological word of the NP it combines with.

d. Within that phonological word, it behaves morphophonemically exactly the same way as the two regular -s inflectional affixes (plural of noun and third singular of finite verb).

(138) **The Syntax of Possessive -’s**

a. $\sigma_1$ is a term constructor of arity NP<sub>ess</sub>; NP<sub>neut</sub>.

This says that -’s combines with any essable NP to produce an NP which can be either a subject or an object but cannot in turn have -’s attached to it.

b. $\sigma_2$ is a term constructor of arity NP<sub>ess</sub>; NP<sub>npro/spec</sub>N.

This says that -’s combines with any essable NP to produce a determiner (i.e. something which in turn can take an N as specifier, thereby forming a nonpronominal (and therefore essable) NP)

(139) **The Phonology of Possessive -’s**

For $\sigma \in \{\sigma_1, \sigma_2\}$:

$$\vdash \forall_x \text{phon}(\sigma(x)) = \text{affix-s}(\text{phon}(x))$$

where $\text{affix-s} : \text{Phon} \supset \text{Phon}$ is subject to:

$$\vdash \forall_{p,q}((q = \epsilon) \supset (\text{affix-s}(p \prec q) = p \prec \text{affix-s}(q)))$$

[Here $x : \text{NP}_{ess}$ and $p, q : \text{Phon}$.]

These axioms ensure that the -’s affixes to the rightmost phonological word of the NP.
(140) Remarks on the Analysis of -'s

a. \(\sigma_1\) and \(\sigma_2\) are our first nonlogical rules, i.e. term constructors other than the introduction and elimination rules for the logical connectives of the syntactic TLC.

b. Note these are unary: -'s is introduced ‘syncategorematically’.

c. Nonlogical axioms of the syntax-phonology interface get the affix onto the last phonological word of the NP\(_{ess}\).

d. There is no evident advantage introducing a form (constant of the syntactic TLC) for -'s. (Cf. Hockett’s notion of “item-and-process”.)

e. To specify the correct form of the affix on phonological words, a theory of segmental phonology is needed that represents phonological words as strings of phonemes and knows about relevant predicates (“phonological features”) on the type Phoneme (the relevant ones here are voiced, coronal, and fricative.)

(141) Semantically Vacuous Prepositions in HPSG

a. HPSG distinguishes from other prepositions the ones that do not have a predicative or locative meaning, but rather function to mark the complements of certain verbs (depend on, accuse ... of) and predicative adjectives (crazy about, fond of, partial to).

b. These are analyzed as selecting an NP\(_{acc}\) complement and no subject.

c. That the prepositional objects are complements is suggested by their extractability (preposition stranding).

d. For example, depend selects as complement a PP\(_{on}\).

e. Technically, a PP\(_{on}\) is a saturated form with HEAD value \(\text{prep}[\text{PFORM }\text{on}, \text{PRD }-]\).

(142) Semantically Vacuous Prepositions in HOG, I

a. Slavishly imitating HPSG, we could posit a basic form type PP for PPs “headed by” semantically vacuous prepositions.

b. HPSG’s PF\(_{FORM}\) values on, of, etc. would be treated as PP-predicates, so the complement selection for depend would still be written PP\(_{on}\).

c. Semantically vacuous on itself would be given as on1 : PP\(_{on}/\text{COMP.NP}_{acc}\).
(143) **Semantically Vacuous Prepositions in HOG, II**

a. But there does not seem to be any reason for the grammar to make reference to a *type* of semantically vacuous PP!
b. Instead we posit separate basic form types OnP, OfP, etc.
c. Then *depend* selects an OnP complement.
d. And semantically vacuous *on* is ON₁ : ONP/COMP NPacc.
e. This analysis reflects the idea that, synchronically, there is no connection between semantically vacuous *on* and the homophonous locative preposition.
f. This obviates the need for the PFORM and PRD features, at least in the analysis of prepositions. (We’ll come back to predicatives.)

(144) **English Subject-Verb Agreement Facts**

- Modals and non-copula past-tense verbs take any NPnom.
- Present-tense non-copula verbs have distinct third-singular and non-third singular forms.
- The copula is special because:
  - Past tense is *was* for first-singular and third-singular, *were* for all other subjects.
  - Present tense is *am* for first-singular, *is* for third-singular, and *were* for all other subjects.

(145) **English Subject-Verb Agreement Analysis I**

- Add NP-predicates 1s and 3s
- Define NP-predicates sg == def 1s ∨ 3s, pl == def ∼ sg
- Revise the lexical entries in (136) as follows:
  
  1 : NPnom ∧ 1s
  he, she : NPnom ∧ 3s
  we, they : NPnom ∧ pl
  it, mine3s, . . . : NP neut ∧ 3s
  you, minepl, . . . : NP neut ∧ pl
  Fido, Felix, Mary, . . . : NPnpro ∧ 3s
(146) **English Subject-Verb Agreement Analysis II**

- Types for subjects of selected finite verbs as follows:
  
  \[
  \begin{align*}
  \text{walks, is} & \rightarrow \text{NP}_{\text{nom} \wedge 3s} \\
  \text{walk} & \rightarrow \text{NP}_{\text{nom} \wedge \sim 3s} \\
  \text{am} & \rightarrow \text{NP}_{\text{nom} \wedge 1s} \\
  \text{was} & \rightarrow \text{NP}_{\text{nom} \wedge \text{sg}} \\
  \text{are, were} & \rightarrow \text{NP}_{\text{nom} \wedge \text{pl}}
  \end{align*}
  \]

- The 's term constructor \( \sigma_1 \) has to be split into two, with value types \( \text{NP}_{\text{neut} \wedge 3s} \) and \( \text{NP}_{\text{neut} \wedge \text{pl}} \) respectively:
  
  \( \text{Mary's is/are green.} \)

(147) **English Subject-Verb Agreement Analysis III**

- As far as subject-verb agreement is concerned, \( \text{you} \) is plural.
- Agreement of NP antecedents with reflexive pronouns is handled by a different mechanism (HOG analog of HPSG’s index features), e.g.
  
  \( \text{You are kidding yourself/yourselfs.} \)

(148) **English Determiner-Noun Agreement Facts I**

In colloquial American varieties:

- Some determiners take only count nouns, some take only mass, some take only plural, some take only singular (count or mass), and some take mass or plural but not count.
- A determiner-noun combination takes third-singular verb agreement if the noun is singular, and non-third singular verb agreement if the noun is plural.

(149) **English Determiner-Noun Agreement Facts II**

- most (all?) count nouns can be coerced into mass nouns:
  
  \( \text{There was chipmunk all over the front porch.} \)

- Most (all?) mass nouns have separate lives as count nouns that refer to a kind:
  
  a. \( \text{This wine will never come out of the carpet.} \)
  
  b. \( \text{This wine will never win any prizes.} \)
English Determiner-Noun Agreement Analysis

- Add N-predicates \text{mas} (mass), \text{cnt} (count), and \text{pln} (plural noun), and define the N-predicate \text{sgn} (singular noun) by \text{sgn} = \text{def cnt} \lor \text{mas}.

- Determiner lexical entries:
  
  \begin{itemize}
  \item much, little, less, all\text{m}, most\text{m}, more\text{m} : \text{NP}_{\text{npro} \land \text{3s}} / \text{SPECN}_{\text{mas}} \\
  \item one, a, each, every : \text{NP}_{\text{npro} \land \text{3s}} / \text{SPECN}_{\text{cnt}} \\
  \item this, that, the\text{s}, some\text{s}, no\text{s}, my\text{s} : \text{NP}_{\text{npro} \land \text{3s}} / \text{SPECN}_{\text{sgn}} \\
  \item two, these, those, many, few, all\text{p}, most\text{p}, more\text{p}, the\text{p}, some\text{p}, no\text{p}, my\text{p} : \text{NP}_{\text{npro} \land \text{pl}} / \text{SPECN}_{\text{pln}}
  \end{itemize}

Two Concerns about Determiner-Noun Agreement

- Many determiners have singular vs. plural entries. Is it possible/desirable to eliminate the duplication?
- It seems an accident that the verb agreement of the NP co-varies with the number of the N.$^3$

Where do Bare Plural NPs Come from? I

- Usually (since Carlson 1977) it is assumed that plural Ns are interpreted as properties (perhaps of aggregate or group individuals) and bare plural NPs as kinds.$^4$
- We can posit a unary rule $\delta$ of arity N$_{\text{pln}}$; \text{NP}_{\text{npro} \land \text{pl}}$.
- $\delta$ is a nonlogical rule (like the two for - 's).
- Semantically, $\delta$ must map a property to a kind.
- So $\delta$ can’t be a subtype embedding, since those don’t affect meaning.

Where do Bare Plural NPs Come from? II

Alternatively, we can posit a phonologically null determiner d : \text{NP}_{\text{npro} \land \text{pl}} / \text{SPECN}_{\text{pln}} that has essentially the same semantic effect (mapping properties to kinds).

Is there a reason to favor one of these two accounts?

---

$^3$G. Webelhuth, p.c.

$^4$In the familiar semantic sense, not in the sense of a recursively defined set of types.
(154) **Verb Form in HPSG**

- vform is a head feature appropriate feature for the verb subtype of head.
- Possible values include fin (finite), inf (infinitive), bse (base), prp (present participle), psp (past participle), and pas (passive participle).

(155) **Verb Form in HOG**

- fin, inf, bse, prp, psp, and pas are treated as S-predicates.
- Syncretism is possible, e.g. between bse and psp:

  The Dean already has, and the faculty wish they could, set [result type $S_{psp/bse}$] the agenda for the coming year.
- Complement selection for nonfinite verbs is just as for their finite counterparts.

(156) **The Problem with Subjects of Nonfinite Verbs I**

a. Consider a finite verb, e.g. ran, that selects an NP$^{nom}$ subject.

b. What kind of subject should the nonfinite forms select?

c. Why not just look at nonfinite sentences and see what kinds of subjects they have?

d. Because it’s hard to find clearcut cases where we can say ‘This is a nonfinite S’ as opposed to an NP followed by a nonfinite NP\S.

e. For example, in ‘raising to object’ sentences like

Kim believes Sandy to admire Chris.

does believes have one complement (the GB analysis) or two (most non-GB analyses)?

(157) **The Problem with Subjects of Nonfinite Verbs II**

a. The clearest cases of nonfinite clauses are present-participial and passive-participial absolutive clauses:

i. **His horse shot out from under him**, Black Bart headed for the Oklahoma line on foot.

ii. **His water supply running low**, Black Bart had to risk descending from the cover of the ponderosa forest.

b. Unfortunately such clauses resist pronominal subjects:
i. (*He/*Him) pursued by lawmen from four jurisdictions, Black Bart headed for the Oklahoma line on foot.

ii. (*He/*Him) getting low on water, Black Bart had to risk descending from the cover of the ponderosa forest.

c. For now we give the subject selection of such verbs as simply NP.

d. We’ll need to revisit this question when we analyze raising.

(158) Passive in HPSG

- In HPSG passive is usually handled by a lexical rule (LR) that maps past participles to passive participless, rearranging the valence in the well-known way.
- HPSG LRs are essentially nonbranching syntactic rules whose mother and daughter are both lexical (Meurers).
- That analysis doesn’t transfer readily to HOG, since
  - in HOG, being lexical means NOT being produced by a tectogrammatical rule; and
  - in HOG, tectogrammatical rules can’t tell whether the things they apply to are lexical or not, since form classes can contain both words and phrases.

(159) Metarules in HOG

- In HOG, if LRs are desired, the straightforward way to get them is not as nonbranching tectogrammatical rules, but rather as operations (‘metarules’) that recursively generate the lexicon (the set of constants whose types are form types) from a set of “basic” constants.
- These are somewhat like the metarules of GPSG, and even more like the metarules of Pregroup Grammar.
- Whether or not obtained via a lexical rule, the correspondence between passive and active forms in HOG is illustrated in (160).
(160) Passive in HOG

a. Some past participles:
   - bit: (NP\ subj S pas)/COMP NP acc
   - given: (NP\ subj S pas)/COMP (NP acc ° NP acc)
   - believed: (NP\ subj S pas)/COMP S
   - told: (NP\ subj S pas)/COMP (NP acc ° S)
   - bet: (NP\ subj S pas)/COMP (NP acc ° NP acc ° S)

b. Corresponding passive participles:
   - bit: (NP\ subj S pas)/COMP ByP
   - given: (NP\ subj S pas)/COMP (NP acc ° ByP)
   - believed: (NP\ subj S pas)/COMP ByP
   - told: (NP\ subj S pas)/COMP (S ° ByP)
   - bet: (NP\ subj S pas)/COMP (NP acc ° S ° ByP)

(161) Passive Semantics in HOG

For each \(n\), where \(n\) is the number of complements the active (past participle) verb \(v\) selects, if \(\text{pas}(v)\) is the corresponding passive participle, then

\[
\text{sem}(\text{pas}(v)) = \lambda_{x_0, \ldots, x_{n-1}} \lambda_{x_n} \text{sem}(v)(x_n, x_0, \ldots, x_{n-2})(x_{n-1})
\]

This is just 1970’s Montague Grammar technology.

(162) HOG Passive Semantics Example

a. \(\vdash \text{sem(bitten}_\text{psp}) = \lambda_y \lambda_x \text{bite}'(x, y)\)
   \(\vdash \text{sem(bitten}_\text{pas}) = \lambda_y \lambda_x \text{bite}'(y, x)\)

b. \(\vdash \text{sem(given}_\text{psp}) = \lambda_y \lambda_z \lambda_x \text{give}'(x, y, z)\)
   \(\vdash \text{sem(given}_\text{pas}) = \lambda_y \lambda_z \lambda_x \text{give}'(z, x, y)\)

c. \(\vdash \text{sem(bet}_\text{psp}) = \lambda_y \lambda_z \lambda_x \text{bet}'(x, y, z, w)\)
   \(\vdash \text{sem(bet}_\text{pas}) = \lambda_y \lambda_z \lambda_x \text{bet}'(w, x, y, z)\)

Semantically, by-passive cyclically permutes the arguments.
(163) **Auxiliaries and Inversion in HPSG**

a. HPSG handles auxiliaries and inversion with two binary head features \( AUX \) and \( INV \) that are appropriate for the type \( verb \).

b. Inverted auxiliaries (which must be finite) are \([AUX +, INV +]\); 
c. uninverted auxiliaries are \([AUX +, INV −]\); and 
d. all other verbs are \([AUX −, INV −]\).

e. Competing syntactic analyses of inversion:
   i. Inverted auxiliaries have same valence as uninverted counterparts; a ‘flat’ rule schema that allows only inverted auxiliary heads makes both subject and complement sisters of the head.
   ii. Inverted auxiliaries select no subject. Instead, what is traditionally called the subject is the least oblique complement, and inverted sentences are licensed by the head-complement schema.

(164) **Interrogatives in HOG**

- There is a distinct basic type \( Q \) for interrogative sentences.
- There are \( Q \)-predicates \( dir \) and \( indir \) for direct and indirect questions.
- We define:
  
  \[
  pdir = \text{def } dir \land \neg \text{ indir} \\
  pind = \text{def } \neg dir \land \text{ indir} \\
  \text{neutq } = \text{def } dir \land \text{ indir}
  \]

(165) **Examples of Interrogatives in HOG**

a. \textit{whether}-questions and uninverted \textit{wh}-questions where the \textit{wh}-phrase is not (contained in) the subject of the question are \( Q_{pind} \):
   - whether Felix is good
   - who Fido bit

b. inverted questions are \( Q_{pdir} \):
   - Is Felix good?
   - Who did Fido bite?

c. \textit{wh}-questions where the \textit{wh}-phrase is (contained in) the subject of the question are \( Q_{neutq} \):
   - Who bit Felix?
   - Whose dog bit Felix?
(166) **Some Lexical Entries**

- that : \(S_{\text{SPEC}S_{\text{fin}}}\)
- whether : \(Q_{\text{pind}/\text{SPEC}S_{\text{fin}}}\)
- wondered, asked : \((\text{NP}_{\text{nom}}\setminus\text{SUBJ}S_{\text{fin}})/\text{COMP}Q_{\text{indir}}\)

(167) **Auxiliaries and Inversion in HOG**

a. There are no features aux and inv.

b. lexical entries for nonfinite and/or uninverted auxiliaries are similar to those of other raising-to-subject verbs, e.g (temporarily ignoring the fact that raised subjects need not be NPs):

- should : \((\text{NP}_{\text{nom}}\setminus\text{SUBJ}S_{\text{fin}})/\text{COMP}(\text{NP}_{\text{nom}}\setminus\text{SUBJ}S_{\text{bse}})\)
- have\textsubscript{bse} : \((\text{NP}\setminus\text{SUBJ}S_{\text{bse}})/\text{COMP}(\text{NP}\setminus\text{SUBJ}S_{\text{psp}})\)
- to : \((\text{NP}\setminus\text{SUBJ}S_{\text{inf}})/\text{COMP}(\text{NP}\setminus\text{SUBJ}S_{\text{bse}})\)

c. inverted auxiliaries are analyzed as per the HPSG subject-as-complement analysis, but with the result type of a purely direct question:

- should\textsubscript{inv} : \(Q_{\text{pdir}}/\text{COMP}(\text{NP}_{\text{nom}} \circ (\text{NP}_{\text{nom}}\setminus\text{SUBJ}S_{\text{bse}}))\)

d. \((\text{SUBJ Fido (should (have\textsubscript{bse} barked\textsubscript{psp} \text{COMP}) \text{COMP}))}\)

e. \((\text{should}\textsubscript{inv} (\text{Fido} \cdot (\text{have\textsubscript{bse} barked\textsubscript{psp} \text{COMP}) \text{COMP}))}\)

(168) **Pleonastics in HPSG**

The feature npform, appropriate for type noun, distinguishes “normal” NPs from pleonastic it ([npform it]) and there ([npform there]).

(169) **Pleonastics in HOG**

There is no clear reason to call pleonastics NPs, so HOG posits distinct basic types It and There:

- it\textsubscript{pleo} : It
- there\textsubscript{pleo} : There

(170) **Some HOG Lexical Entries Referring to Pleonastics**

- rained, snowed, hailed : It\setminus\text{SUBJ}S_{\text{fin}}
- appeared\textsubscript{extra}, seemed\textsubscript{extra}, transpired : \((\text{It}\setminus\text{SUBJ}S_{\text{fin}})/\text{COMP}S\)
- is\textsubscript{exist} : \((\text{There}\setminus\text{SUBJ}S_{\text{fin}})/\text{COMP}\text{NP}_{\text{nproA3s}}\) [ignoring for now the fact that only certain quantified NPs can appear]
(171) **Equi in HPSG**
In HPSG, equi verbs (and adjectives):

- select complements which select NP subjects; and
- the controller (subject or least oblique complement) is constrained by the lexical entry to be coindexed with the complement subject.

(172) **Equi in HOG**
In HOG, equi verbs (and adjectives):

- select complements which select NP subjects, as in HPSG.
- The status of indices in HOG is unsettled, but we can account for equi-type entailments 1970’s-style (by meaning postulates).

(173) **Subject-Equi Semantics Example**

a. hopes\(_1\) : (NP\(_{\text{nom} \wedge 3s \backslash \text{SUBj} \text{S} \text{fin}}\))/COMP\(\text{S} \text{fin}\)

\[\vdash \text{sem}(\text{hopes}_1) = \lambda p \lambda x \text{hopes}'(x, p)\]

Kim hopes she wins.

b. hopes\(_2\) : (NP\(_{\text{nom} \wedge 3s \backslash \text{SUBj} \text{S} \text{fin}}\))/COMP(NP\(_{\backslash \text{SUBj} \text{S} \text{inf}}\))

\[\vdash \text{sem}(\text{hopes}_2) = \lambda f \lambda x \text{hopes}'(x, f(x))\]

Kim hopes to win.

(174) **Raising in HPSG**
In HPSG, raising verbs (and adjectives):

- select complements which select subjects, but don’t care what the subjects are
- the controller (subject or least oblique complement) is constrained by the lexical entry to have its SYNSEM value shared with that of the complement subject.
(175) **HOG Raising**
In HOG, raising verbs (and adjectives):

- select complements which select subjects, as in HPSG
- the syntactic effect of raising is obtained by type-schematizing
  the lexical entries, with two occurrences of the same type
  metavariable, one in the controller position and one in the
  complement subject position.
- Notational convention: suppose \( a : A \supset \text{Bool} \) is an \( A \)-predicate,
  and \( B \) a type. Then \( B_a \) means \( A_a \) if \( B \) is \( A \), but simply
  means \( B \) otherwise.

(176) **Raising to Subject Example**

a. \( \text{seemed}_{\text{extra}} : (\text{It}_{\text{subj}}S_{\text{fin}})/\text{COMP}S \)  
   \( \vdash \text{sem}(\text{seemed}_{\text{extra}}) = \lambda p. \lambda x. \text{seem}^\prime(p) \)  
   It seemed Kim was sad.

b. \( \text{seemed}_{2,A} : (A_{\text{nom}}S_{\text{fin}})/\text{COMP}(A_{\text{nom}}\backslash \text{SUBJ}S_{\text{inf}}) \)  
   \( \vdash \text{sem}(\text{seemed}_{2,A}) = \lambda f : \text{Sem}(A) \supset \text{Prop} \lambda x : \text{Sem}(A) \text{seem}^\prime(f(x)) \)  
   i. She seemed to be sad. \([A = \text{NP}]\)
   ii. It seemed to be windy. \([A = \text{It}]\)
   iii. There seemed to be a problem. \([A = \text{There}]\)

(177) **Raising to Object Example**

a. \( \text{proved}_1 : (\text{NP}_{\text{nom}}S_{\text{fin}})/\text{COMP}S_{\text{fin}} \)  
   \( \vdash \text{sem}(\text{proved}_1) = \lambda p. \lambda x. \text{prove}^\prime(x, p) \)  
   Kim proved \( \pi \) is irrational.

b. \( \text{proved}_{2,A} : (\text{NP}_{\text{nom}}\backslash \text{SUBJ}S_{\text{fin}})/\text{COMP}(A_{\text{acc}} \circ (A_{\text{acc}}\backslash \text{SUBJ}S_{\text{inf}})) \)  
   \( \vdash \text{sem}(\text{proved}_{2,A}) = \lambda y : \text{Sem}(A), f : (\text{Sem}(A) \supset \text{Prop}) \lambda x : \text{Ind} \text{prove}^\prime(x, f(y)) \)  
   i. Kim proved him to be irrational. \([A = \text{NP}]\)
   ii. Kim proved it to be risky to trust Sandy. \([A = \text{It}]\)
   iii. Kim proved there to be an irregular prime. \([A = \text{There}]\)
Complements of *Be*

In the PSG tradition, expressions that can be complements to the copula *be* are called **predicatives**. These include the following five categories (with informal abbreviations):^5

a. $\text{VP}_{\text{prp}}$ (present-participial verb phrase)
   
   Black Bart was making a run for the Oklahoma line.

b. $\text{VP}_{\text{pas}}$ (passive-participial verb phrase)
   
   Pecos Slim was pursued by lawmen from four different jurisdictions.

c. $\text{AP}_{\text{prd}}$ (predicative adjective phrase)
   
   Buckshot Pete was desperately low on funds.

d. $\text{PP}_{\text{prd}}$ (predicative prepositional phrase)
   
   Nohorse Bob was on his one-eyed mule.

e. $\text{NP}_{\text{prd}}$ (predicative noun phrase)
   
   Tumbleweed Jack was the meanest hombre north of the Rio Grande.

Just One *Be*

Though other traditions often refer to “equative *be*”, “predicative *be*”, “passive *be*”, etc., coordination facts suggest examples like those in (178) all involve the same *be*:

a. Black Bart was making a run for the Oklahoma line and pursued by lawmen from four different jurisdictions.

b. Tumbleweed Jack was desperately low on funds and the meanest hombre north of the Rio Grande.

c. Nohorse Bob was on his one-eyed mule and desperately low on funds.

d. Pecos Slim was the meanest hombre north of the Rio Grande and making a run for the Oklahoma line.

Nonverbal predicatives are VP-like in being able to combine with something to the left to express a proposition:

- a. i. His mule was too tired to go on.
  
    ii. His mule TOO TIRED TO GO ON, Nohorse Bob headed for the Oklahoma border on foot.

- b. i. It was already too hot to go on.

---

^5 Another category sometimes included are the so-called **modal infinitives**, e.g. *the Dean is to address the full faculty at 4:00*. But these are of a rather formal register and do not coordinate comfortably with the other types, so we set them aside here.
ii. it already TOO HOT TO GO ON, Black Bart dozed in the shade of a cactus and waited for evening to fall.

c. i. His sidekick Buckshot Pete was a crack shot and an able cook.
   ii. His sidekick Buckshot Pete A CRACK SHOT AND AN ABLE COOK, Tumbleweed Jack was able to focus on planning the robberies and counting the money.

d. i. His canteen was now at the bottom of the gulch.
   ii. His canteen NOW AT THE BOTTOM OF THE GULCH, Pecos Slim had to head into the high desert without water.

(181) Nonverbal Predicatives can be RTO Complements

a. i. Sheriff McGraw considered Tumbleweed Jack to be the meanest hombre north of the Rio Grande.
   ii. Sheriff McGraw considered Tumbleweed Jack THE MEANEST HOMBRE NORTH OF THE RIO GRANDE.

b. i. Sheriff McGraw considered the high desert north of Sidewinder Gulch to be under his jurisdiction.
   ii. Sheriff McGraw considered the high desert north of Sidewinder Gulch UNDER HIS JURISDICTION.

c. i. Sheriff McGraw considered it to be too hot for the posse to continue the pursuit.
   ii. Sheriff McGraw considered it TOO HOT FOR THE POSSE TO CONTINUE THE PURSUIT.

d. i. Sheriff McGraw considered Pecos Slim to be too mean to tangle with single-handed.
   ii. Sheriff McGraw considered Pecos Slim TOO MEAN TO TANGLE WITH SINGLE-HANDED.

(182) Pronouns/Anaphor complements of nonverbal predicatives act just as if they were objects of verbs:

a. i. Kim₁ thinks Sandy₂ admires/loves him₁/₂.
   ii. Kim₁ thinks Sandy₂ admires/loves himself₁/₂.

b. i. Kim₁ thinks Sandy₂ is fond of/crazy about him₁/₂.
   ii. Kim₁ thinks Sandy₂ is fond of/crazy about himself₁/₂.

c. i. Kim₁ thinks Sandy₂ is behind/above him₁/₂.
   ii. Kim₁ thinks Sandy₂ is behind/above himself₁/₂.
d. i. Kim\textsubscript{1} thinks Sandy\textsubscript{2} is a parody/an imitation/the opposite of him\textsubscript{1/2}.

ii. Kim\textsubscript{1} thinks Sandy\textsubscript{2} is a parody/an imitation/the opposite of himself\textsubscript{1/2}. [himself\textsubscript{1} okay as a correction]

(183) **Subjectless absolutives**

a. READY TO ANYTHING TO PLACATE THE SCREENING COMMITTEE, Butch was sure to be admitted to the secret society.

b. A BIG DONOR TO ALL THE RIGHT SCHOOLS, Butch was sure to be admitted to the secret society.

c. ON THE BOARDS OF NINE FORTUNE 500 COMPANIES, Butch was sure to be admitted to the secret society.

d. EXERTING VAST INFLUENCE IN THE CITADELS OF POWER, Butch was sure to be admitted to the secret society.

e. CONSIDERED A MAJOR-LEAGUE MOVER AND SHAKER, Butch was sure to be admitted to the secret society.

(184) **Absolutives with subjects**

a. ((What) with) his aunt Buffy READY TO DO ANYTHING TO PLACATE THE SCREENING COMMITTEE, Butch was sure to be admitted to the secret society.

b. ((What) with) his aunt Buffy A BIG DONOR TO ALL THE RIGHT SCHOOLS, Butch was sure to be admitted to the secret society.

c. ((What) with) his aunt Buffy ON THE BOARDS OF NINE FORTUNE 500 COMPANIES, Butch was sure to be admitted to the secret society.

d. ((What) with) his aunt Buffy EXERTING VAST INFLUENCE IN THE CITADELS OF POWER, Butch was sure to be admitted to the secret society.

e. ((What) with) his aunt Buffy CONSIDERED A MAJOR-LEAGUE MOVER AND SHAKER, Butch was sure to be admitted to the secret society.

(185) **Concessive (Though-)adverbials**

Though ALLERGIC TO CATS/A LIFELONG VEGAN/IN TEXAS/RUNNING FOR OFFICE/HOUNDED BY CREDITORS, Dana did what was expected of him.
(186) **So-called “Depictive” adverbials**  
Kim came back from Texas READY FOR BUSINESS/A REPUBLICAN/IN A BOXCAR/ DRIVING A BENTLEY/ PURSUED BY LAWMEN FROM FOUR STATES.

(187) **Complements of see/regard/view ... as**  
I don’t see/view/regard him as REASONABLE/A QUALIFIED CANDIDATE/READY FOR PRIME TIME/TAINTED BY THE SCANDAL/HELPING OUR CAUSE.

(188) **While-temporal adverbials**  
While SICK/A GRAD STUDENT/OUT OF WORK/LOOKING FOR A JOB/ INVOLVED WITH SEVERAL HOPELESS CAUSES, Chris became addicted to blue M&M’s.

(189) **Locative/Predicative Inversion**

a. Predicative PPs:
   i. BESIDE HIM sat a very fat Malay chewing tobacco.
   ii. BENEATH THIS was another table commemorating the manifold virtues of the Parkers’ daughter.
   iii. AWAY ON THE HILL stood a figure he could only see as a silhouette.

b. Predicative APs:
   i. ALSO BENEFICIAL was the camaraderie.
   ii. EVEN MORE IMPORTANT is convenient and reliable service.
   iii. CONSPICUOUSLY ABSENT was Mary,

c. Predicative NPs:
   i. ALSO REPUBLICANS are Ohio’s two senators, Mike DeWine and George Voinovich.
   ii. A FURTHER CONSEQUENCE OF THE AXIOM OF CHOICE is the theorem that every infinite boolean algebra has a nonprincipal ultrafilter.

d. Present participial VPs:
   i. QUICKLY SLIDING TOWARD THEM was the stack of 4’ x 8’ plywood that moments before appeared to be resting securely on the top shelf.
ii. CROSSING THE ROAD AHEAD was a doe that appeared to have become disoriented.

e. Passive participial VPs:
   i. PASSED BY A VOICE VOTE was a resolution thanking Tom Delay for meritorious service.
   ii. NARROWLY DEFEATED was a proposal to amend the Constitution to define marriage as being between a man and a woman.
   iii. LIKewise SHOWN TO BE INDEPENDENT OF ZF was the Generalized Continuum Hypothesis.

(190) If we exclude predicative NPs . . .

   . . . an even greater range of environments predicatives becomes available:

   a. Optional second complement in there constructions:
      There is a unicorn FAST ASLEEP/IN THE GARDEN/STUFFED WITH KAPOK/SLEEPING.
   b. Postnominal modifiers (“reduced relatives”):
      A unicorn THAT FLAMBOYANT/IN A RED TUTU/WEARING BLACK FISHNETS/ STUFFED WITH KAPOK is seldom seen in these parts.

(191) Obvious Analytic Gambit:

   • Since nonverbal predicative phrases seem to be just like VPs (except for having nonverbal heads), let’s simply analyze them as “nonverbal VPs”.
   • In HOG, a “VP” is an $A_{\text{Subj}}S_a$ where $A$ is a type of subject and $a$ is a “verb form value”, i.e. one of the S-predicates fin, inf, bse, prp, psp, or pas.
   • So we simply posit new S-predicates adj, prep, and prdp.

(192) HOG Predicative Categories

   a. $S_{\text{adj}}$ means “adjectival small clause”
   b. $S_{\text{prep}}$ means “prepositional small clause”
   c. $S_{\text{prdp}}$ means “predicative nominal small clause”
   d. $NP_{\text{Subj}}S_{\text{adj}}$ means “predicative AP that takes a ‘normal’ subject”
e. $S_{adj}$ means “predicative PP that takes pleonastic it subject” (e.g. weather adjectives)

f. $S_{prep}$ means “predicative PP”

g. $S_{prdnp}$ means “predicative NP”

(193) **Auxiliary be in HOG**

- The HOG replacement for G/HPSG’s boolean head feature $PRD$ (appropriate for the subtype substantive of head) is the defined S-predicate

  \[ PRD = \text{def} \ prp \lor \text{pas} \lor \text{adj} \lor \text{prep} \lor \text{prdnp} \]

- Uninverted auxiliary be (base form) is

  \[ be_A : (A_{Sbse})/\text{comp}(A_{Sprd}) \]

(194) **Unfinished Business**

All three nonverbal predicative categories have nonpredicative counterparts:

a. nonpredicative (i.e. prenominal, or attributive) adjectives

b. nonpredicative (yet semantically nonvacuous) prepositions

c. ordinary nonpredicative NPs

So far we’ve said nothing that relates the nonverbal predicatives to their nonpredicative counterparts.

(195) **Predicative vs. Attributive Adjectives—Facts**

a. Many predicative adjectives lack attributive counterparts, e.g. the class of two-syllable a-adjectives including adrift, ajar, aloft, amiss, askew, adrift, awry, etc.

b. Many attributive adjectives lack predicative counterparts, e.g. alleged, presumed, former, future, mere, utter, outright, etc.

c. Attributive adjective phrases must be head-final (and so cannot have complements or post-head modifiers).

d. Some adjectives have attributive vs. predicative meanings that are not relatable to each other in any systematic way, e.g. a through street vs. we’re through; a ready answer vs. we’re ready; a complete idiot vs. the proof is complete.
Predicative vs. Attributive Adjectives—Analysis

The connection between attributive and predicative adjectives has to be accounted for lexically, not syntactically:

\[
\begin{align*}
\text{complete}_{att} & : N / \text{LMOD} N \\
\text{complete}_{prd} & : \text{NP} \setminus \text{SUBJ} \text{S}_{adj}
\end{align*}
\]

Predicative vs. Nonpredicative Locative Prepositions—Facts

a. Usually locative PPs are treated as inherently predicative:

\[
\begin{align*}
(\text{man in Chicago})' = \lambda x (\text{man}'(x) \land \text{in}'(x, \text{Chicago}')) \\
(\text{die in Chicago})' = \lambda x \exists e (\text{die}'(x, e) \land \text{in}'(e, \text{Chicago}'))
\end{align*}
\]

I.e. \text{in}'(x, y) predicates that the \text{in}' relation holds between \(x\) and \(y\).

b. But any locative preposition can also be used nonpredicatively to express a function from individuals to locations:

i. Under the bed is Lydia’s favorite hiding place.

ii. Sputnik prefers behind the couch to under the bed.

iii. Rick, what about after Paris?

Predicative vs. Nonpredicative Locative Prepositions—Analysis

a. TLC/HOL doesn’t seem to provide any way to derive the meanings of nonpredicative locative PPs from their predicative counterparts.

b. But the other direction is easy: map location \(l\) to the individual property \(\lambda x \text{located-at}(x, l)\).

c. Conclusion: we can treat the relationship between nonpredicative locative PPs (say, PP_{loc}) and predicative PPs by a unary rule (non-logical term constructor) of arity PP_{loc}; NP \setminus \text{SUBJ} \text{S}_{prep}.

Predicative vs. Nonpredicative NPs

a. There may be semantic factors that make it hard to imagine contexts for certain (especially downward-entailing) quantificational NPs as predicatives.

b. But there do not seem to be lexical exceptions to the empirical hypothesis that any N can be used in the formation of NPs that can be used predicatively.

c. Possibly extravagant claim: any NP can be used predicatively.
d. So we will analyze predicative NPs by another unary rule (nonlogical term constructor) of arity NP; NP\SUBJ S\S_{\text{prdp}}.

e. Semantically, this rule is translated as a function that maps individual (concept) \( y \) to the individual property \( \lambda_x \text{equals}(y, x) \).\(^6\)

(200) **Semantic Analysis of Auxiliary \textit{be}**

a. Recall the lexical entry for auxiliary \textit{be}:
\[
\text{be}_A : (A\SUBJ \text{Subbase})/\text{COMP}(A\SUBJ \text{Subpred})
\]

b. The corresponding semantic type is:
\[
(\text{Sem}(A) \supset \text{Prop}) \supset (\text{Sem}(A) \supset \text{Prop})
\]

c. Leaving aside the interpretation of tense, we can simply assume that the meaning of \textit{be} is “apply the property expressed by the predicative complement to the (possibly dummy) sense expressed by the subject; i.e.
\[
\vdash \text{sem}(\text{be}_A) = \lambda_P \lambda_x P(x)
\]

d. This is just the curried version of the function application combinator \( \lambda_P \lambda_x P(x) \).

(201) **An Exercise**

Prove that \textit{Fido is Rover} means \textit{equals}(\text{Fido'}, \text{Rover'}).

Hint 1: the complement is not \textit{Rover} : NP, but rather \textit{pnp}(\text{Rover}) : NP\SUBJ S_{\text{prdp}}, where \textit{pnp} is the unary rule (term constructor) that makes NPs predicative.

Hint 2: The meaning of \textit{pnp} is given by the axiom
\[
\vdash \forall_x [\text{sem}(\text{pnp}(x)) = \lambda_y \text{equals}(y, x)].
\]

N.B: In Hyperintensional Semantics, \textit{equals} : (\text{Ind} \land \text{Ind}) \supset \text{Bool} is subject to the meaning postulate
\[
\vdash \forall_{w, x, y} [\text{ext}_w (\text{equals}(x, y)) = (\text{ext}_w(x) = \text{ext}_w(y))]
\]

This says that a pair of individual concepts has the \textit{equals} property at a world iff the two individual concepts have the same extension at that world.

\(^6\)Caveat: \textit{equals} is \textit{not} the HOL constant \( \equiv_{\text{Ind}} \), since that is of type (\text{Ind} \land \text{Ind}) \supset \text{Bool}, whereas \textit{equals} is of type (\text{Ind} \land \text{Ind}) \supset \text{Prop}. The connection is described in hyperintensional semantics by a meaning postulate:
\[
\vdash \forall_{w, x, y} [\text{ext}_w (\text{equals}(x, y)) = (\text{ext}_w(x) = \text{ext}_w(y))]
\]

That is, \textit{equals} is the property that holds of a pair of individual concepts at a world just in case their extensions at that world are one and the same entity.
A Perhaps Surprising Consequence

It is the meaning of the predicative-NP term constructor, NOT the meaning of the copula, that gives *Fido is Rover* an equational meaning.

Many Copulas

Earlier we said there was only one copula (not counting different inflected forms, and not counting the subject-auxiliary-inverted forms).

But what we really meant was that there is only one ‘canonical’ copula. There must also be several ‘noncanonical copulas’ in order to handle such phenomena as:

Some noncanonical copula phenomena:

a. There is no beer. [Plain Existential]
b. There is no beer in the fridge. [Existential Predicative]
c. Outside the cafe was a ragtag band of ravenous Tolstoy scholars. [Heavy Inversion]
d. Beside the fountain was Jill. [Predicative Inversion]
e. What he was doing was digging a hole. [Plain Pseudocleft]
f. What he was doing was he was digging a hole. [Amalgamated Pseudocleft]

Some puzzling facts:

a. (c-f) lack Subject-Auxiliary Inversion counterparts.
b. (d-f) cannot be embedded under Raising to Subject.

Predicative Inversion

Pollard and Weibelhuth (in prep.) analyze Predicative Inversion in terms of a lexical rule that applies not only to the (canonical) copula, but to a wide range of ‘ergative’ verbs (including verbs of existence, spatial orientation, directed motion, etc.).

a. **Input type:** \((\text{NP}_{\text{subj}}S)/\text{comp-}A\)
   
   where \(A\) is some subtype of predicative (i.e. \(\text{NP}_{\text{subj}}S_{\text{prd}}\))

b. **Output type:** \((A_{\text{top}}S)/\text{comp-NP}\)
Example of Predicative Inversion Lexical Rule

a. **Input:** was : (NP\_{SUBJ}S\_{fin})/COMP(NP\_{SUBJ}S\_{prd})

b. **Output:** was\_{predinv} : ((NP\_{SUBJ}S\_{prd})\_{TOP}S\_{fin})/COMP-NP

- The semantic effect of the LR is just to permute the two arguments.
- Predicate inversion does not admit Subject-Auxiliary Inversion or Raising to Subject because the precopular constituent is not a subject.
- Other properties of predicate inversion arise from information-structural and other pragmatic properties linked to the **TOPIC** grammatical relation.

What’s the Problem?

Earlier we tentatively specified the subjects of nonfinite verbs (and predicatives) (e.g. *leave*\_{bse}) as simply NP, with no reference to case. To see why this is problematic consider:

a. i. I should leave.
   ii. Mary heard me leave.

b. i. **should**\_NP : (NP\_{nom}\_{SUBJ}S\_{fin})/COMP(NP\_{nom}\_{SUBJ}S\_{bse})
   ii. **heard\_2**\_NP : (NP\_{nom}\_{SUBJ}S\_{fin})/COMP(A\_{acc} o (A\_{acc}\_{SUBJ}S\_{bse}))

The raising verbs seek nonfinite VP complements that seek subjects with a specific case, but *leave*\_{bse} seeks merely an NP subject.

The Solution: Domain Restrictors

a. In HOL with subtypes, if \(A\) and \(B\) are types and \(a\) is an \(A\)-predicate, there is a combinator \(\rho_{A,B,a} : (A \supset B) \supset (A_a \subset B)\). In an interpretation \(I\) it is interpreted as a function that applies to functions from \(I(A)\) to \(I(B)\) and restricts their domains to the subset \(I(A_a)\):

\[
\rho_{A,B,a} = \text{def} \lambda f \in A \supset B \lambda x \in A_a f(\text{embed}_a(x))
\]

b. We can’t define analogs of the \(\rho\) combinators for ‘valence-flavored’ implicative types (e.g. \(A\_{\text{\textbackslash SUBJ}}B\)) since there are no lambda-binders of these flavors (e.g. there is no \(\lambda^{\text{\textbackslash SUBJ}}\)).

c. Instead we simply introduce a type-indexed family of term constructors that do the same thing:

\(|_{A,B,a}\) is a term constructor of arity \(A\_{\text{\textbackslash SUBJ}}B; A_a\_{\text{\textbackslash SUBJ}}B\).

d. if \(f : A\_{\text{\textbackslash SUBJ}}B\), we usually write simply \(f\_{a}\) instead of \(f\_{|A,B,a}\).
(208) **The Case of Raised NPs**

a. The complement of (206a-i) *I should leave* is not leave\textsubscript{bse} but rather leave\textsubscript{bse, nom}, which seeks a subject of type NP\textsubscript{nom}.

b. The complement of (206a-ii) *Mary proved me to be a liar* is not leave\textsubscript{bse} but rather leave\textsubscript{bse, acc}, which seeks a subject of type NP\textsubscript{acc}.

(209) **A Problem with Disjunctive Subcategorization**

a. The verb became can take either predicative AP or predicative NP complements:

b. i. John became conservative.

ii. John became a Republican.

c. One way to handle this is by assuming became is ambiguous between a verb that takes an NP\textsubscript{nom} \SUBJ\textsubscript{S} \adj and one that takes an NP\textsubscript{nom} \SUBJ\textsubscript{S} \prdn.

d. But this can’t be the right solution, because of examples like

   John became conservative and a Republican.

(210) **Disjunctive Subcategorization, Take 2**

a. Alternatively, we could give a single lexical entry for became that specifies its complement as NP\textsubscript{nom} \SUBJ\textsubscript{S} \adj \prdn.

b. But now we have a new problem: became cannot take conservative as a complement because conservative is an adjectival predicative, but become only knows to look for predicatives whose result type is *either* adjectival *or* nominal!

(211) **Another Solution: Codomain Extenders**

a. In HOL with subtypes, if \(A\) and \(B\) are types, and \(a\) is a \(B\)-predicate, there is a combinator \(\eta_{A,B,a} : (A \supset B) \supset (A \subset B)\). In an interpretation \(I\) it is interpreted as a function that applies to functions from \(I(A)\) to \(I(B)\) and extends their codomains to \(I(B)\):

\[
\eta_{A,B,a} = \text{def } \lambda f \in A \supset B, a \lambda x \in A \text{ embed}_a (f(\langle x \rangle)).
\]

b. We can’t define analogs of the \(\eta\)-combinators for ‘valence-flavored’ implicative types (e.g. \(A \setminus \SUBJ B\)) since there are no lambda-binders of these flavors (e.g. there is no \(\lambda^{\setminus \SUBJ}\)).
c. Instead we simply introduce a type-indexed family of term constructors that do the same thing:

\[ \eta_{A,B,a} \] is a term constructor of arity \( A_{\text{subj}}B_a, A_{\text{subj}}B \).

d. if \( f : A_{\text{subj}}B_a \), we usually write simply \( a \mid f \) instead of \( \eta_{A,B,a}(f) \).

(212) Codomain Extension for Disjunctive Subcategorization
In (209b-i) \textit{John became conservative}, the complement \textit{conservative} is the form obtained thus:

\begin{itemize}
  \item a. start with a lexical \( \text{NP}_{\text{subj}}\text{S_{adj}} \);
  \item b. apply an \( \eta \)-constructor that extends its codomain to \( \text{S_{adj}V_{prdn}} \);
  \item c. apply the constructor \( \rho_{\text{nom}} \) to restrict the domain to \( \text{NP}_{\text{nom}} \).
\end{itemize}

(213) Disjunctive Subcategorization and Coordination I

\begin{itemize}
  \item Our analysis of \textit{became} above still does not take care of \textit{John became conservative and a Republican}, since we don’t yet have an analysis of coordination!
  \item As a first attempt, let’s try adapting the standard CG analysis. We start by introducing a type-indexed family of lexical entries \( \text{and}_A : (A_{\text{lconj}}A)_{/\text{rconj}}A \).
  \item Here \( \text{lconj} \) and \( /\text{rconj} \) are new flavors of implication for left and right conjuncts respectively.
  \item \( A \) ranges over (some subset of the) form types.
\end{itemize}

(214) Disjunctive Subcategorization and Coordination II

\begin{itemize}
  \item To handle \textit{John became conservative and a Republican}, we’ll try imitating the standard CG analysis of coordination of unlikes (due to Morrill, and to Bayer and Johnson).
  \item Start with \textit{conservative} : \( \text{NP}_{\text{subj}}\text{S_{adj}} \) and \( \text{pnp(\text{embed}_{\text{npro}_\text{\&3s}}(a \text{ Republican }^{\text{spec}}))} : \text{NP}_{\text{subj}}\text{S_{prdn}} \).
  \item Apply appropriate codomain extenders \( \eta_1 \) and \( \eta_2 \) respectively to convert both of these to forms of type \( \text{NP}_{\text{subj}}\text{S_{adj}V_{prdn}} \).
  \item Now we have two phrases of the same category, so we can conjoin them using \( \text{and}_{\text{S_{adj}V_{prdn}}} \).
\end{itemize}
(215) **A Fly in the Ointment**

Unfortunately the analysis just proposed renders the grammar theory inconsistent! (It was all those ands that did it.)

- We analyzed *conservative and a Republican* as an NP\subj(S\adj\prdnp).
- Giving this a subject (say *his father*) produces an S\adj\prdnp, which is the same as an S\adj \cup S\prdnp.
- So the small clause *his father conservative and a Republican* must be the embedded image of either (i) an S\adj, or (ii) an S\prdnp.
- But obviously it cannot be either one.
- The original CG analysis suffers from a version of this same problem (see Pollard and Hana 2003).

(216) **A New Approach to Coordination**

a. The intuition is this: if you’re hungry and either a bagel or a muffin would do, you’d probably also settle for the breakfast special consisting of a bagel followed by a muffin.

b. Likewise, *became* isn’t really insisting on either a predicative NP or a predicative AP; it would settle for a ‘breakfast special’ (i.e. a coordinate structure) of one and then the other.

c. That is, predicates usually analyzed as subcategorizing for a disjunction are less demanding than they are usually thought to be.

d. For implicative types, being less demanding amounts to having a larger antecedent type.

e. So the category for a coordinate structure whose left and right conjuncts are *A* and *B* respectively isn’t *A \cup B*, but something bigger.

f. Let’s call that bigger type G(*A \cup B*) (a generalized *A \cup B*).
Further Properties of Category Generalization

a. For any category $C$, a coordinate structure whose conjuncts are both generalized $Cs$ should also be a generalized $C$.

b. If $C$ is a subtype of $D$, then the category of generalized $Cs$ should be a subcategory of the category of generalized $Ds$.

c. In terms of the set-theoretic interpretation of the grammar, these observations can be summarized by saying that $G$ is interpreted as a closure operation on sets, i.e. it is (a) increasing; (b) monotone; and (c) idempotent.

d. In terms of the logic of syntactic types, category generalization is reminiscent of a possibility (diamond) modal operator.

Category Generalization Formalized

We introduce:

- a unary type constructor $G$ (Generalization)
- term constructors $\text{gen}_A$ of arity $A;G(A)$ (so any $A$ also ‘counts as’ a $G(A)$)
- term constructors $\text{gen}'_A$ of arity $G(G(A));G(A)$ (so any generalized $G(A)$ also ‘counts as’ a $G(A)$)
- In terms of category theory, this machinery characterizes category generalization as a monad.

A New Analysis of Coordination

- Conjunctions and, or have their own category Conj.
- We introduce a new family of (ternary) term constructors (rules) $\kappa_A$ of arity $G(A),\text{Conj},G(A);G(A)$.
- $\kappa_A$ combines two generalized $As$ with a conjunction in between to form a new generalized $A$.

The Last Word on Coordination . . . for Today

a. What $became$ subcategorizes for is not what we said in (210), viz. $NP_{nom}\backslash_{\text{SUBJ}}S_{adj\lor\text{prednp}}$, but rather $G$ of that!

b. Informally: $became$ wants as its complement either (i) a predicative that is either adjectival or nominal, or else (ii) a coordinate structure built up from such things.

c. This is just the beginning of a general theory of coordination.
(221) **What are Nonlocal Phenomena?**
Pretheoretical answer: relationships between forms (or positions within forms) that can extend across clause boundaries in a sentence, or across sentences in discourse. Examples:

a. “Á-movement”, aka “extraction”
b. Right Node Raising
c. Comparative Subdeletion
d. Embedding of a Quantificational NP within the clause where it takes scope
e. “Binding” between a pronoun and a “distant” antecedent
f. Pied Piping: Embedding of a relative or interrogative pronoun within a form that is required to contain one
g. Embeddings of an *in situ* *wh*-expression within the question where it takes scope
h. Embedding of a comparative word within the phrase to which the *than*-phrase that it licenses attaches.

(222) **“Á-Movement”: The Filler-Gap Connection**
Gaps in Á-constructions can be embedded arbitrarily deeply relative to where the raised right node is attached:

a. Topicalization
   i. BAGELS₁, I like *t*₁.
   ii. BAGELS₁, I think Kim likes *t*₁.

b. Constituent Questions
   i. WHICH BAGELS₁ do you like *t*₁?.
   ii. WHICH BAGELS₁, do you think Kim likes *t*₁?

c. Relative Clauses
   i. These are the bagels (WHICH/THAT)₁ I like *t*₁.
   ii. These are the bagels (WHICH/THAT)₁ I think Kim likes *t*₁.

d. Clefts
   i. It is these bagels THAT₁ I like *t*₁.
   ii. It is these bagels THAT₁ I think Kim likes *t*₁.

e. Pseudoclefts
   i. WHAT₁ I like *t*₁ is bagels.
   ii. WHAT₁ I think Kim likes *t*₁ is bagels.
(223) **Right Node Raising**
  Gaps in Right Node Raising constructions can be embedded arbitrarily deeply relative to where the raised right node is attached:
  
a. Felix bit $t_i$, and Sylvester scratched $t_i$, a mean old Rottweiler.
b. Kim thinks Felix bit $t_i$, but Sandy denies Sylvester scratched $t_i$, a mean old Rottweiler.

(224) **Comparative Subdeletion**
  A subdeleted specifier (“trice”) in a than- or as-phrase can be arbitrarily deeply embedded inside it:
  
a. More people came THAN $d_i$, tickets were sold.
b. More people came THAN the vendor reported that $d_i$, tickets were sold.
c. Twice as many people came AS $d_i$, tickets were sold.
d. Twice as many people came AS the vendor reported that $d_i$, tickets were sold.

(225) **Quantifier Scope**
  The semantic scope of a quantificational NP can be
  
a. narrower than surface position would indicate:
    i. A UNICORN seems $t_i$ to be in the garden. [It seems that there is a unicorn is in the garden, but maybe there isn’t.]
    ii. A unicorn seems to be in the garden. [There is a unicorn which seems to be in the garden.]
  b. wider than surface position would indicate:
    i. Q $Fido$ has a bone in [every $t_i$, corner of the room].
    ii. Fido has a (Stegosaurus) bone which is in every corner of the room.
    iii. Q Some lunatic in [every $t_i$, little midwestern town] would like to blow it off the face.
    iv. Some lunatic who works in every little midwestern town would like to blow every last one of them off the face.
“Binding Principle B”
The “antecedent” of a nonreflexive pronoun must be suitably “distant” from it:

a. Kim_i thought Sandy_j shaved him_{ij/sj}.
b. Kim_i persuaded Sandy_j to shave him_{ij/i}.
c. Kim_i promised Sandy_j to shave him_{ij/si}.
d. Kim_i’s father_j shaved him_{ij/sj}.
e. Kim_i liked the picture of him_i.
f. The picture of him_i pleased Kim_i.

Pied Piping
The relative pronoun can be deeply embedded in the “extracted” constituent in a relative clause:

a. These are the reports which_i the government prescribes the height of the lettering on the covers of t_i.
b. These are the reports [the covers of which_i] the government prescribes the height of the lettering on t_i.
c. These are the reports [the lettering on the covers of which_i] the government prescribes the height of t_i.
d. These are the reports [the height of the lettering on the covers of which_i] the government prescribes t_i.

Multiple Wh-Questions
Multiple wh-questions can contain arbitrarily deeply embedded in situ wh-expressions, which:

a. have scope over the whole question:
   i. Who ordered what?
   ii. Who said Fido ate what?
   iii. Who thinks who said Fido ate what?
b. or even have scope over a “bigger” question:
   i. Who knows who ate what? [Kim knows who ate what.]
   ii. Who knows who ate what? [Kim knows who ate the bagels, and Sandy knows who ate the muffins.]
Comparative Dependencies
A comparative form can be arbitrarily deeply embedded relative to
where the than- or as-phrase it licenses attaches:

a. Kim thought that more people came than tickets were sold. [Kim
thought that the number of people that came exceeded the number
of tickets that were sold.]

b. Kim thought that more people came than tickets were sold. [The
number of people that Kim thought came exceeded the number of
tickets that were sold.]

c. Kim thought that as many people came as tickets were sold. [Kim
thought that the number of people that came was the same as the
number of tickets that were sold.]

d. Kim thought that as many people came as tickets were sold. [The
number of people that Kim thought came was the same as the
number of tickets that were sold.]

HOG Analysis of Nonlocal Dependencies: Basics

a. Each kind of nonlocal dependency (cf. (221)) corresponds to a dif-
fferent flavor of implication (in the syntactic type logic).

b. Unlike the flavors of implication associated with valence features,
the nonlocal flavors of implication lack elimination (“Merge”) rules.

c. “Bottoms” of nonlocal dependencies (traces, RNR gaps, trices, wh-
words, etc.) are different flavors of hypotheses.

d. Some of the nonlocal implications have introduction rules (with as-
associated flavors of lambdas), corresponding to quantifier/operator
binding in GB or to-bind/retrieve features in HPSG.

e. Pronominal “binding” is not really binding (implication introduc-
tion) but rather instances of the structural rule of contraction.

f. So called “parasitic gaps” are also instances of contraction.

g. Undischarged hypotheses (e.g. deictic pronouns, comparative forms
without than or as phrases) are logical parameters, and have to be
interpreted by anchoring in the discourse context.

TLC Revisited
To motivate the technical aspects of our treatment of nonlocal
phenomena, we first look at a reformulation of Implicative TLC
that makes explicit some aspects of how hypotheses are managed
that were left implicit before.
(232) Implicative TLC the Old Way

a. For each type $A$, there are variables $x_0^A, x_1^A, \ldots : A$

b. For each type, there can be constants of that type

c. Application (Modus Ponens):
   if $f : A \supset B$ and $a : A$, then $f(a) : B$.

d. Abstraction (Hypothetical Proof):
   if $x : A$ is a variable and $b : B$, then $\lambda x \ b : A \supset B$.

N.B.: Each type has its own set of variables (Church typing).

(233) Contexts

- A context is a string of variable/type pairs, written to the left of the turnstile at each node of the proof tree.
- The contexts keep track of the undischarged hypotheses.
- Contexts are strings (not just sets or lists) because we track
  a. the order of hypotheses
  b. multiple occurrences of the same hypothesis.
- We make explicit the structural rules that allow contexts to be restructured.
- Instead of typed variables (Church typing), we use a fixed stock of general-purpose variables and let the contexts track what types are assigned to the variables in a given proof (Curry typing).
- No context can assign distinct types to the same variable.
- We use capital Greek letters as metavariables over contexts.

(234) Implicative TLC Reformulated Using Contexts

**Hypotheses:** $x : A \vdash x : A$

**Nonlogical Axioms:** $\vdash a : A$ ($a$ a constant of type $A$)

**Modus Ponens:** if $\Gamma \vdash f : A \subset B$ and $\Delta \vdash a : A$
then $\Gamma, \Delta \vdash f(a) : B$

**Hypothetical Proof:** if $x : A, \Gamma \vdash b : B$
then $\Gamma \vdash \lambda x b : A \supset B$
Weakening: if $\Gamma, \Delta \vdash b : B$ then $\Gamma, x : A, \Delta \vdash b : B$

Permutation: if $\Gamma, x : A, y : B, \Delta \vdash c : C$
then $\Gamma, y : B, x : A, \Delta \vdash c : C$

Contraction: if $\Gamma, x : A, x : A, \Delta \vdash b : B$
then $\Gamma, x : A, \Delta \vdash b : B$.

(235) Note Especially Hypothetical Proof:
If $x : A, \Gamma \vdash b : B$, then $\Gamma \vdash \lambda_x b : A \supset B$

• Another name for this is Implication Introduction
• This is the usual way to prove a conditional: hypothesize the antecedent, prove the consequent, then withdraw the hypothesis.

(236) Structural Rules I
The last three rules in (234, top of this page) are called structural rules.

• Weakening lets you add a redundant hypothesis, which can be subsequently discharged (vacuous lambda abstraction), to prove a conditional where the antecedent is ‘irrelevant’ to the consequent.
• Permutation means the order of the hypotheses can be ignored.
• Contraction means duplicate hypotheses can be ignored.

(237) Structural Rules II
• Usually hypotheses are thought of as a set not a string, so Permutatation and Contraction are implicit.
• But the string way generalizes to our situation, where there are hypotheses (for nonlocal dependencies) for multiple flavors of implication, each of which admits a different subset of structural rules.
Substructural Implicative Logics

The propositional logic (PL) axiomatized by Implicative TLC is Implicative Intuitionistic PL. We get different PLs by discarding different subsets of structural rules, e.g.

- Relevant PL: discard Weakening
- Ticket PL: discard Weakening and Permutation
- Linear PL: discard Weakening and Contraction
- Lambek Calculus has left and right flavors of implication each with its own Modus Ponens and Hypothetical Proof, and no structural rules.

When Hypotheses Come in Flavors

- Extended to handle nonlocal dependencies, our grammar logic will have multiple flavors of implication each of which manages hypotheses in different ways,
- So we partition the contexts to track different flavors of hypotheses.
- In this tutorial we only have time to look at one flavor of nonlocal dependency, the A-Movement flavor.
- With a nod to G/HPSG, we call the corresponding flavor of implication simply / (pronounced “slash”).

HOG Syntax as it Stands

Lexicon: Optional words of each form type.

Complement Fusion: If $a : A$ and $b : B$
then $a \cdot b : A \circ B$

Subject Merge: If $a : A$ and $f : A\text{\_SUBJ}\ B$
then $(\text{\_SUBJ} a \ f) : B$

Complement Merge: If $f : B/\text{\_COMP}\ A$ and $a : A$
then $(f \ a \text{\_COMP}) : B$

Specifiee Merge: If $f : B/\text{\_SPEC}\ A$ and $a : A$
then $(f \ a \text{\_SPEC}) : B$

Subtype Embedding: If $a : A \supset \text{Bool}$ and $b : A_a$
then $\text{\_Embed}_a(b) : A$

Domain Restriction: If $a : A \supset \text{Bool}$ and $f : A\text{\_SUBJ}\ B$
then $f|a : A_a\text{\_SUBJ} B$ (likewise other implications)
**Codomain Extension:** If $a : B \supset \text{Bool}$ and $f : A \setminus \text{subj} B_a$ then $a \mid f : A \setminus \text{subj} B$ (likewise other implications)

**Generalization:** If $a : A$ then $\text{gen}_A(a) : \text{G}(A)$

**Generalization':** If $a : \text{G}(G(A))$ then $\text{gen}'_A(a) : \text{G}(A)$

**Coordination:** If $a : \text{G}(A)$, $b : \text{Conj}$, and $c : \text{G}(A)$, then $\kappa_A(a, b, c) : \text{G}(A)$

**Nonlogical unary rules:** for '-'s, bare plurals, predicative NPs, and predicative locative PPs.

(241) Reformulating HOG with Contexts

- For now we are concerned only with SLASH-flavored hypotheses.
- None of the unary rules affect contexts, i.e. the context of the conclusion (mother) is the same as for the premiss (daughter).
- The remaining rules are reformulated as below.

(242) HOG Syntax with Contexts I

**Complement Fusion:** If $\Gamma \vdash a : A$ and $\Delta \vdash b : B$ then $\Gamma, \Delta \vdash a \cdot b : A \circ B$

**Subject Merge:** If $\Gamma \vdash a : A$ and $\Delta \vdash f : A \setminus \text{subj} B$ then $\Gamma, \Delta \vdash (\text{subj} a f) : B$

**Complement Merge:** If $\Gamma \vdash f : B / \text{comp} A$ and $\Delta \vdash a : A$ then $\Gamma, \Delta \vdash (f a \text{comp}) : B$

**Specific Merge:** If $\Gamma \vdash f : B / \text{spec} A$ and $\Delta \vdash a : A$ then $\Gamma, \Delta \vdash (f a \text{spec}) : B$

**Coordination:** If $\Gamma \vdash a : \text{G}(A)$, $b : \text{Conj}$, and $\Gamma \vdash c : \text{G}(A)$, then $\Gamma \vdash \kappa_A(a, b, c) : \text{G}(A)$

(243) Coordination is Special

- Rules except Coordination pass up the SLASH hypotheses of the daughters in the order in which they are temporally realized, just like SLASH-Inheritance in G/HPG.
- Coordination differs from all other rules in requiring that the contexts of the coordinated daughters coincide with the context of the whole coordinate phrase.
- In linear logic jargon, coordination is additive relative to SLASH, and the other rules are multiplicative.
(244) **Across the Board (ATB) Condition**
Empirically, the additivity of Coordination corresponds to the Across-the-Board (ATB) Condition on extraction from coordinate phrases:

a. BAGELS\textsubscript{i}, Kim likes \textit{t\textsubscript{i}} and Sandy hates \textit{t\textsubscript{i}}.
b. *BAGELS\textsubscript{i}, Kim likes muffins and Sandy hates \textit{t\textsubscript{i}}.
c. *BAGELS\textsubscript{i}, Kim likes \textit{t\textsubscript{i}} and Sandy hates muffins.

(245) **Additional Rules for slash**

**Finite Move:** If \( x : A, \Gamma \vdash s : S\text{fin} \)
then \( \Gamma \vdash \lambda_x^{\text{slash}} s : S\text{fin}/A \)

**Infinitive Move:** If \( x : \text{NP}\text{acc}, \Gamma \vdash v : B/S\text{inf} \)
then \( \Gamma \vdash \lambda_x^{\text{slash}} v : (B \setminus \text{subj} S\text{inf})/\text{NP}\text{acc} \)

**Topicalization:** If \( \Gamma \vdash a : A \) and \( \Delta \vdash b : S\text{fin}/A \)
then \( \Gamma, \Delta \vdash \tau(a, b) : S\text{top} \)

**Contraction:** If \( \Gamma, x : A, x : A, \Delta \vdash b : B \)
then \( \Gamma, x : A, \Delta \vdash b : B \)

(246) **Analysis of Topicalized Sentences**

- We need a new S-predicate \text{top} for topicalized sentences since their distribution is slightly different than ordinary finite sentences:
  a. I wonder whether Kim likes muffins.
  b. *I wonder whether muffins, Kim likes.

- Finite Move allows binding of a trace (undischarged slash-hypothesis) of type \text{NP}\text{acc} in a finite S (e.g. \textit{Kim likes t}), to form an \( S\text{fin}/\text{NP}\text{acc} \).

- Topicalization can combine an \text{NP}\text{acc} (e.g. \textit{muffins}) with this to form the \( S\text{top} \textit{Muffins, Kim likes} \).

- Other rules similar to Topicalization form finite constituent questions, finite relative clauses, cleft clauses, and pseudocleft clauses.
(247) Analysis of ‘Tough-Movement’

- Infinitive Move allows binding of an accusative NP trace inside an ‘infinitive VP (NP\_{SUBJ}S\_inf) to form a (NP\_{SUBJ}S\_inf)/NP\_acc, e.g. to please t.
- This is just the kind of phrase that ‘tough’-predicative adjectives, e.g. easy, take as their complements to form ‘predicative APs’ (NP\_{SUBJ}S\_adj), e.g. easy to please.
- The lexical semantics of the adjective handles the semantic connection between the subject and the trace.
- Infinitive VPs with bound NP\_acc-traces also occur in purpose adjuncts, infinitive relatives, infinitive constituent questions, too/enough-constructions, etc.

(248) Analysis of ‘Parasitic Gaps’

- Two traces can be ‘co-bound’:
  a. This book is hard impossible to start t without finishing t.
  b. These are the reports we filed t without reading t.
  c. Which rebel leader did rivals of t assassinate t?
- We analyze these as cases where Contraction applies before Move.
- This analysis contradicts an empirical hypotheses proposed with GB (the Bijection Principle), which says there must be a one-to-one correspondence between traces and binders).

(249) The Prohibition on Crossed Dependencies

a. i. This violin\_j, even the most difficult sonatas are easy \_i [to play t\_i on t\_j].
   ii. *This sonata\_i, even the most exquisitely crafted violins are difficult \_j [to play t\_i on t\_j].

b. i. Which problems, don’t you know who\_j [to talk to t\_j about t\_i]?
   ii. Which people\_j don’t you know what\_i [to talk to t\_j about t\_i]?

These facts follow directly from the lack of a structural rule of Permutation for the slash-implication. (They’d be fine if there was one.)
(250) **The Prohibition on ‘Movement from Nowhere’**

b. *Who did you see Kim?
c. *The student who I talked to Sandy just arrived.
d. *What this country needs a good five-cent cigar is a good 20-cent
draft beer.
e. *It’s Kim that Sandy likes Dana.
f. *John is easy to please Mary.

These facts follow directly from the lack of a structural rule of
Weakening for the SLASH-implication. (They’d be fine if there
was one.)

(251) Á-Movement Summary

The basic facts about Á-Movement follow from the following prop-
erties of SLASH-implication:

- It is a ticket implication (i.e. it admits Contraction but not
  Weakening or Permutation)
- There is not a general Hypothetical Proof schema, but only
certain instances of it (finite S and infinitive VP)
- Coordination is additive with respect to SLASH-implication,
other rules are multiplicative.

(252) Where Do Traces Come From?

- The straightforward approach is to add the schema
  **Hypotheses:** $x : A \vdash x : A$
- But this overgenerate wildly:
  a. *Saw, I t the cat.
  b. *The, I saw t cat.
  c. *Cat, I saw the t.
  d. *Marmalade, I saw the t cat.
  e. *Frege was Italian, I wonder whether t.
- We need a mechanism, analogous to GB’s Empty Category
  Principle or HPSG’s Trace Principle, that constrains which
rules traces can be premisses to.
- We leave the precise formulation for another day.