Using the Jape Natural Deduction Tutorial

0.1 Finding Jape

If you have an account on the Linguistics Department computer system, log in and type “jape” at the command prompt.

If you do not have such an account, you can go to the site http://jape.org.uk and download onto your computer the jape package that is appropriate for your operating system (linux, Solaris, Mac OS, Windows)—which of course you may choose to do even if you do have a ling account. Unpack the .jar archive and follow instructions.

0.2 Using Jape

After starting Jape, you will see Jape’s “control window”. Under the File menu, choose “Open New Theory...”, and when prompted, select the file (on ling)

/opt/local/lib/jape/examples/jnj/jnj.jt

You will then see a new window called “Conjectures”. You can select one of the arguments in the window, then click on “Prove”, and you will get another window called “Proof # 1”, with the argument’s premises and conclusions at the bottom: premises and conclusion are separated by “…”.

0.3 Some Notational Details:

1. The symbol \( \Rightarrow \) is used for the conditional (“\( \rightarrow \)” in PtMW), NOT “logically implies.” Similarly “\( \Leftrightarrow \)” is used for the biconditional.

2. The symbol “\( \vdash \)” is used for “logically implies”: so “Formula, \( \ldots \)Formula_2” refers to an argument where “Formula, \( \ldots \)” are the premises and Formula_2 is the conclusion. A single formula in the Conjectures window that has no \( \vdash \) is to be proved from no premises at all.

3. The JNJ system uses false for a tautology (a bad choice of terminology—“\( \perp \)” should have been used), and true for a tautology.
0.4 Rules of Inference

The ones you can use in JNJ are those on the accompanying handout “An Alternative Formulation of Natural Deduction …”; take a careful look at that if you haven’t already (more on these below). In Jape, you can see the list of rules by pulling down the “Propositional Rules” menu.

To use a rule of inference: first select a line from the proof in progress by clicking on it with Mouse-1 (this outlines it in red); to select a second line, click Shift-Mouse-1. Then pull down the “Propositional Rules” menu and choose one of the rules to apply to your selected line. If you have selected the lines and rule correctly, Jape will then enter the results into the proof, with the rule name and lines used added to the right of the conclusion; if you have not selected appropriate lines for this rule, Jape will complain and give you some hints as to what your error was (but these are not very helpful usually).

There are two ways to use rules of inference in Jape, “forward” starting with the premises, and “backward” starting from the conclusion. If you select a line or lines that correspond to the premises of some rule of inference, then click on that rule, Jape will add in the conclusion below them. If you click on a line that is the conclusion for some rule, then click on the rule, Jape will add in any needed intermediate steps above the conclusion. In general, the “elimination” rules work forward, and the “introduction” rules work backward (although some rules can be used either way, depending on whether you have selected their premises or conclusion).

For example, if you select a formula of the form \((p \land q)\) and choose “\(\land\)-elim-1”, Jape will enter “\(p\)” just below, with the rule and line used indicated on the right of it. But “\(\land\)-intro”, being an introduction rule, usually must be applied backwards; i.e. you must select a formula of the form \((p \land q)\), choose \(\land\)-intro, and Jape will write \(p\) and \(q\) above it.

To prove a conjecture, what you need to do is work from one or both ends of the proof toward the middle, i.e. toward the “…”; when the two parts meet, the “…” will disappear and the proof is complete.

There are a number of constraints on which rules can be applied under which circumstances, and also on which line or lines must be selected in order to get Jape to apply the rule. (For example, for some backwards applications, you have to select a line above the conclusion as well as the conclusion.) These conditions are best grasped, at this point, by trial and error; often Jape will give you a hint as to why an incorrect attempted application or selection is wrong. Helpful features are that (i) conclusion lines are preceded by \(\vdash\), unproven assumptions are not, and (ii) when you select a line that must be used as a conclusion, the red box is open at the top, but when you select a line that must have to be used as an assumption, the box will be open at the bottom.

If you decide you have made a bad choice of a rule application, you can undo that step by going to the Edit menu and selecting “Undo.” You can select Undo again to erase the previous step, etc.

Lines of a proof that depend on assumptions are enclosed in a box together with those assumptions (the way that PtMW use a vertical line in their proofs).

0.5 Neg-Elim, Neg-Intro, and False-Elim

Where PtMW has a single principle of Indirect Proof (Reductio ad absurdum), Jape, with its more mechanical execution of proofs, requires two separate steps. These are not too intuitive. When both a formula \(\phi\) and
have been derived, “Neg-Elimination” is applied to derive a contradiction, false. Then from false and an assumption \( \phi \) used to derive it, we use “Neg-Intro” to derive \( \neg \phi \). Here’s an example: \( \neg \neg p \vdash p \):

1. \( \neg \neg p \) assumption
2. \( \neg p \) assumption
3. \( \text{false} \) 1,2 \( \neg \)-elim
4. \( p \) 2-3 false-elim

When you want to draw a conclusion of the form \( \neg p \) rather than \( p \) by a reductio proof, you must use the rule false-elim in the last step instead of neg-intro (see other handout for this).

A new inference rule is \( \lor \)-elim (cf. other handout). If you have an assumption \( (p \lor q) \) and want to derive \( r \), then if you can derive \( r \) from \( p \) and can also derive \( r \) from \( q \), then you are justified in concluding \( r \) from \( (p \lor q) \).

### 0.6 Completed Proof

When you have completed a proof (indicated by the fact that there are no . . . left in it, you can “record” the proof, which will let you use it later on as a rule of inference. To do this, you must delete the proof window; when you do, a dialog box will appear asking you whether you want to record the proof. Check yes. Then a checkmark will appear next to the argument in the “Conjectures” box. (Note that saving the proof is not the same thing as recording it. You may want to save it as well, but only if you record it can you use it in further proofs.

To use an argument you have proved in another proof, select (red box) the relevant lines, then go to the Conjectures window, highlight the relevant proved argument, and click “Apply” at the bottom of the window. New lines will appear in your proof. However, they will not contain the atomic formula you actually want, but only “variables” over formulas, indicated with \( a, b, t, u \). (These correspond to \( P, Q, R \) in the propositional equivalences in PtMW; they are “meta-variables”.) What you need to do then is make Jape substitute the formulas you actually want in place of these variables. The means for this is unification (which is, incidentally, an important procedure in computational linguistics). To do this, you first need to text-select the variables you want to be replaced and then text-select the formulas you want to put in place of them. To text-select a formula or part of one (not to be confused with ordinary selection of a formula), put the cursor over the beginning of the part you want and press Control-Mouse-2, then drag the cursor over the part you want: this will appear in yellow. Then do the same in the formula where you want the substitutions to take place. Draw down the “Edit” menu and choose “Unify”. This will make Jape “pattern-match” between the formulas and merge (unify) them—if you have selected the parts correctly.

### 0.7 Suggested practice with Jape

If you want to get started using Jape, I recommend you start first by proving the formulas in the Conjectures window that have only conditionals, such as \( (p \Rightarrow (q \Rightarrow r)) \vdash (q \Rightarrow (p \Rightarrow r)) \). This will get you in the swing of doing proofs easily, and soon you’ll be able to crank them out pretty fast. Next, I suggest you try proofs that include \( \land \) as well as \( \Rightarrow \). Save the ones with \( \neg \) or \( \lor \) for last.