Translating English Noun Phrases

1 Restriction and Nuclear Scope of a Noun Phrase Representing a Quantifier

In class earlier, we introduced some standard translations with quantified variables corresponding to the subject noun phrase of a sentence.

(1) a. All cats are mammals.
   b. \((\forall x)(C(x) \rightarrow M(x))\)

(2) a. Some cats are mammals.
   b. \((\exists x)(C(x) \land M(x))\)

(Recall the discussion in class of why \(\rightarrow\) is used in the first case and \(\land\) in the second.)

It is helpful to identify the subformulas of these translations that correspond to the different constituents of the English sentences. In both cases, \(C(x)\) translates the noun cats, and \(M(x)\) translates the VP are mammals. To capture the general pattern of translation of a NPs, the quantified formula corresponding to it is often described as containing a restriction and a nuclear scope. An English NP of the form every \(N\) corresponds to a template formula

\[(\forall x)[(\ldots x \ldots)_R \rightarrow (\ldots x \ldots)_S]\]

where the first ellipsis is filled by a formula representing the restriction, and the second ellipsis by the nuclear scope. Likewise, an English NP of the form some \(N\) corresponds to a template formula

\[(\exists x)[(\ldots x \ldots)_R \land (\ldots x \ldots)_S]\]

The idea is that the determiner (all, some) determines which quantifier will be used, and the nominal expression (after the determiner) has the effect of restricting the universe of discourse to a subset of individuals that satisfy a particular predicate, namely the restriction. The remainder of the sentence is translated by the nuclear scope, which makes a statement that all (or some) individuals in the restricted subset have some property or participate in some relation. In both (1b) and (2b), the restriction is the formula \(C(x)\) and the nuclear scope is \(M(x)\). (Templates can also be constructed for other determiners like no, not all, the, etc.)

The same pattern of restricted quantification is applicable to noun phrases in other sentence positions, as well as subject NPs.

(3) a. Kim dislikes all wealthy people.
b. \( (\forall x) [(W(x) \land P(x)) \rightarrow D(k, x)] \)

\( W(x) : x \) is wealthy;
\( P(x) : x \) is a person;
\( D(x, y) : x \) dislikes \( y \).

Notice also that in (3), the noun modifier *wealthy* is treated as part of the the template’s restriction section; in particular, it is translated as forming a conjunction with the translation of the head noun. That this is the correct way to symbolize the sentence is by this quasi-logic paraphrase:

(4) The following is true for all \( x \): if \( x \) is a person and \( x \) is wealthy, then Kim dislikes \( x \).

In fact, this applies to ALL modifiers within noun phrases: the modifier can be translated as part of the restriction, but should not be translated as part of the scope.

Thought Problem (or Homework Problem):

The following also seems to be a possible paraphrase of (3a):

(5) The following is true for all \( x \): if \( x \) is a person, then if \( x \) is wealthy, then Kim dislikes \( x \).

which suggests rather that this should be the correct translation of (3):

(6) \( (\forall x)[P(x) \rightarrow (W(x) \rightarrow D(k, x))] \)

Which is correct? Why?

2 Restriction and Scope: some pitfalls

The real value of the restriction-scope format (in symbolizing English sentences in predicate logic) becomes apparent when we symbolize sentences with two quantifiers, such as (7):

(7) A dog chased every cat.

Sometimes students learning logic try to symbolize such a sentence by beginning:

(8) \( (\exists x)(\forall y)[(D(x) \land C(y)) \ ? H(x, y)] \)

and then wonder what connective to write where the “?” is; is it “→” or “&”; the answer, as we shall see, is that there is no way to complete (8) in a way that gives a correct representation of the truth conditions of (7).

If we use the restriction-scope template to construct the translation, then notice it must be used twice, as follows: first, we symbolize the scope-section of the sentence, with variables put in place of the noun phrases; using \( H(x, y) \) for “\( x \) chases \( y \)”, we write:

(9) \( H(x, y) \)

Then we add the translation of the NP *every cat*, using \( C(y) \) as the restriction and (9) as the scope:
(10) \((\forall y)(C(y) \rightarrow H(x, y))\)

Then this translation becomes the scope for the translation of the NP *a dog*, with the universal template and \(D(x)\) as the restriction (notice we use now “&” as the connective):

(11) \((\exists x)[D(x) \& (\forall y)(C(y) \rightarrow H(x, y))]\)

This is in fact a correct translation for (8), though not the only one. The sentence has two readings: one is paraphrasable as “there is a dog which chased every cat” (i.e. a single dog), or “For every cat there is a dog which chases it” – this latter reading allows that different dogs may have chased each cat, whereas the first reading does not. We can derive this second reading by using the same core scope and same two quantifier templates, but adding them in a different order. To the scope translation of the “core” of the sentence in (7) we add first the template for the NP *a dog*:

(12) \((\exists x)[D(x) \& H(x, y)]\)

and then add to this the template for *every cat*:

(13) \((\forall y)[C(y) \rightarrow (\exists x)(D(x) \& H(x, y))]\)

This is the correct symbolization of the other reading of (8). For most people, (13) and (11) are also the easiest symbolizations to recognize as representing (8).

3 Translating English Sentences in Steps

As an aid to translating complicated sentences without getting lost, some logic textbooks recommend converting the English sentence one part at a time; this technique fits together well with the idea of quantificational restriction-scope templates. Consider this example (after I. M. Copi):

(14) Any good amateur can beat some professional

As a first step we can write

\((\forall x)[(x \text{ is a talented amateur}) \rightarrow (x \text{ can beat some professional})]\)

(Notice that this splits up the noun phrase *any talented amateur* into a quantifier template plus a restriction and nuclear scope, which are not yet translated.) We can symbolize the modifier and noun in the restriction, giving this:

\((\forall x)[(T(x) \land A(x)) \rightarrow (x \text{ can beat some professional})]\)

Now the consequent of the formula (i.e. the nuclear scope of the noun phrase we’ve translated), which we can isolate for clarity as

\(x \text{ can beat some professional}\)
can be symbolized as

\[(\exists y)[(y \text{ is a professional}) \land (x \text{ can beat } y)]\]

Plugging in predicate constants in this gives

\[(\exists y)[P(y) \land B(x, y)]\]

and inserting this subformula in the larger one then gives the translation we want:

\[(\forall x)[(T(x) \land A(x)) \rightarrow (\exists y)[P(y) \land B(x, y)]]\]

Notice that the nuclear scope of one noun phrase (any talented amateur) contains in its nuclear scope the translation of the other noun phrase (some professional), which in turn has as its nuclear scope the “core” of the sentence x can beat y. This “nesting” of quantifier phrases is characteristic of a correct translation of an English sentence. Although there may be other, logically equivalent translations that aren’t nested this way, the step-wise method always gives you nested versions if used correctly, and you will soon learn to recognize what a formula “says” far more easily if it is nested this way than if it appears in some other form — such as pre-nex form where all the quantifiers have been moved to the left.

Got the idea? If you think so, you can perform two checks on your understanding at this point:

1. The example sentence is ambiguous: it can also be understood as having wide scope for some professional. Use the step-wise method to come up with the correct translation for this other reading, without getting the restrictions and scopes in the wrong places.

2. Apply the method to this sentence, which has four noun phrases that correspond to quantifiers. (Although more than one scope relationship for the quantifiers can be represented, just construct one translation at this point.)

Anyone who promises everything to everyone is certain to disappoint somebody.

Key: use “H(x)” for “x is a person”, “P(x,y,z)” for “x promises y to z”.  

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