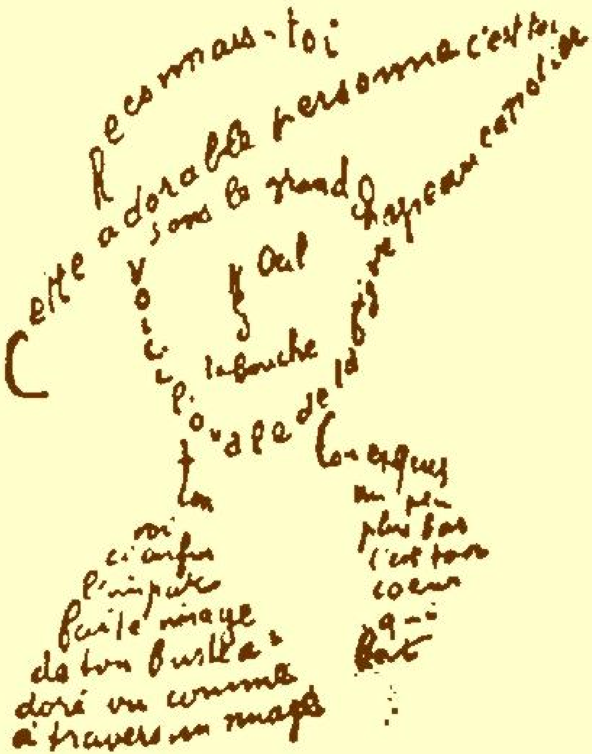
Reconnais-toi
Cette adorable personne c'est toi
sous le grand chapeau canotier
oeil
nez
la bouche
Voici l'ovale de ta figure
ton cou exquis
voici enfin l'imparfaite image de ton buste adoré vu comme à travers un nuage
un peu plus bas c'est ton coeur qui bat

Typing
Binding &
Anaphora

Dynamic Contexts as $\lambda\mu$-Terms

Philippe de Groote

Inria Nancy - Grand Est

Outline

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- A type-theoretic reconstruction of DRT.

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- A type-theoretic view of dynamic logic.

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Challenge:

- to express dynamics using "static" primitives (in particular, to avoid the "destructive assignment" problem, wich necessitates a LISP-like *gensym* operator).

Proposed solution:

- to interpret a sentence according to both its left and right contexts;
- to abstract these two kinds of contexts over the meaning of the sentences.

Typing the left and the right contexts

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Montague semantics is based on Church's simple type theory, which provides a full hierarchy of functional types built upon two atomic types:

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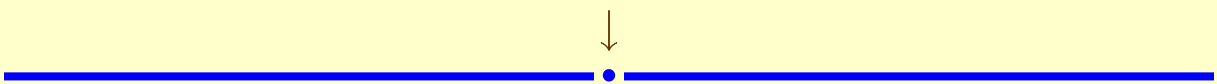

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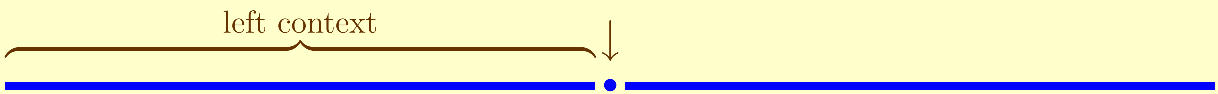

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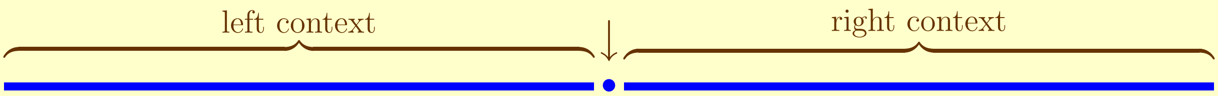

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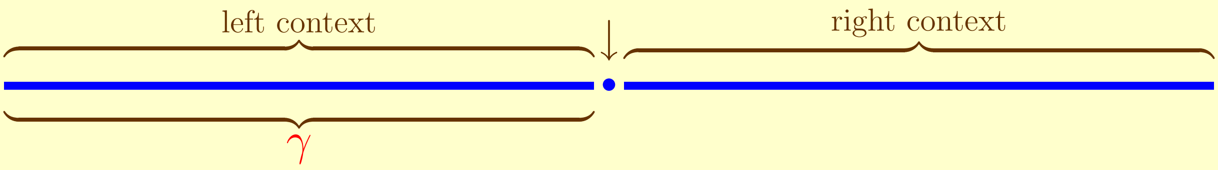

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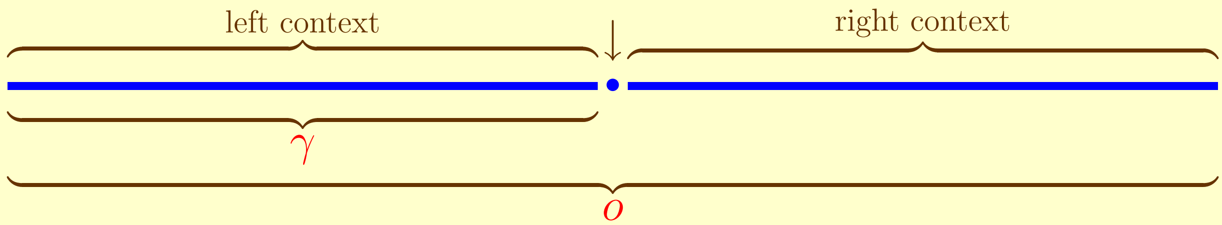

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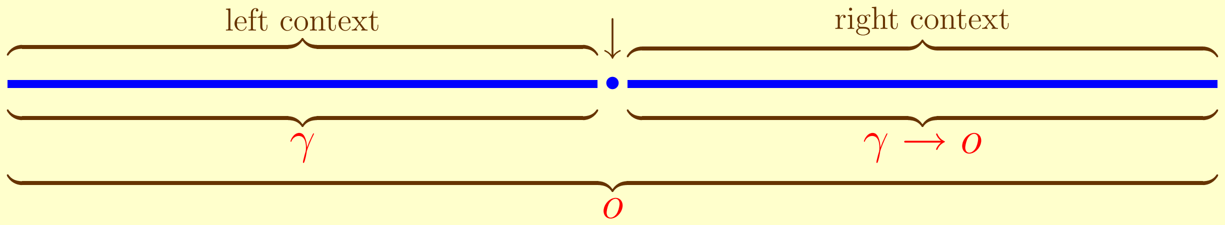

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Composition of two sentence interpretations

$$[\![S_1.\, S_2]\!] = \lambda e \phi.\, [\![S_1]\!]\, e\, (\lambda e'.\, [\![S_2]\!]\, e'\, \phi)$$

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may be rephrased as follows:

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...which might seem a little bit involved.

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We share with DRT the two following assumptions:

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- the main form of quantification is existential (it introduces referential markers).

Consequently, our logic will be based on conjunction and existential quantification (defined as primitives). The other connectives will be obtained using negation (a third primitive) and de Morgan's laws.

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$$\wedge \ : \ o \rightarrow o \rightarrow o \qquad (conjunction)$$

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DYNAMIC PRIMITIVES

$$\begin{array}{rcll} :: & : & \iota \rightarrow \gamma \rightarrow \gamma & (\textit{context updating}) \\ \mathsf{sel} & : & \gamma \rightarrow \iota & (\textit{choice operator}) \end{array}$$

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Negation

We do not want the continuation of the discourse to fall into the scope of the negation. Consequently, negation must be defined as follows:

$$\sim A \triangleq \lambda e \phi .\, \neg\, (A\, e\, (\lambda e .\, \top)) \wedge \phi\, e$$

Implication and Universal Quantification

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These are defined using de Morgan's laws:

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\begin{aligned}
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$$\begin{aligned} \overline{R\, t_1 \, \ldots \, t_n} &= \lambda e \phi.\, R\, t_1 \, \ldots \, t_n \wedge \phi\, e \\ \overline{\neg A} &= {\sim}\overline{A} \\ \overline{A \wedge B} &= \overline{A} \sqcap \overline{B} \\ \overline{\exists x.\, A} &= \Sigma x.\, \overline{A} \end{aligned}$$

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This embedding is such that, for every term e of type γ:

$$A \equiv \overline{A}\, e\, (\lambda e.\, \top)$$

Donkey Sentence Revisited

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Montague-like semantic interpretation:

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$$[\![\text{beats}]\!]\ [\![\text{it}]\!]\ ([\![\text{every}]\!]\ ([\![\text{who}]\!]\ ([\![\text{owns}]\!]\ ([\![\text{a}]\!]\ [\![\text{donkey}]\!]))\ [\![\text{farmer}]\!]))$$

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β-reduces to the following term (modulo de Morgan's laws):

$$\lambda e\phi.\,(\forall x.\,\mathbf{farmer}\,x \supset (\forall y.\,\mathbf{donkey}\,y \supset (\mathbf{own}\,x\,y \supset \mathbf{beat}\,x\,(\mathsf{sel}\,(x{::}y{::}e))))) \wedge \phi\,e$$

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that is, assuming that sel is a "perfect" anaphora resolution operator:

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- Interpreting $(\gamma \to o) \to o$ as a double negation, we may identify it with γ.
- To this end, we must use a calculus whose type system corresponds to classical logic.
- The $\lambda\mu$-calculus is such a calculus.

Call by Value $\lambda\mu$-calculus

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syntax

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The set of $\lambda\mu$-terms is built upon two disjoint sets of variables (namely, λ-variables and μ) according to the following rules:

$$
\begin{array}{rcll}
T & ::= & c & \text{(constant)} \\
 & | & x & \text{(λ-variable)} \\
 & | & (\lambda x.\, T) & \text{(λ-abstraction)} \\
 & | & (T\, T) & \text{(application)} \\
 & | & (\mu a.\, T) & \text{(μ-abstraction)} \\
 & | & a\,(T) & \text{(naming)} \\
 & | & \langle T \rangle & \text{(reset)}
\end{array}
$$

Call by Value $\lambda\mu$-calculus

syntax

The set of $\lambda\mu$-terms is built upon two disjoint sets of variables (namely, λ-variables and μ) according to the following rules:

$$
\begin{array}{rcll}
T & ::= & c & \text{(constant)} \\
& | & x & (\lambda\text{-variable}) \\
& | & (\lambda x. T) & (\lambda\text{-abstraction}) \\
& | & (T\,T) & \text{(application)} \\
& | & (\mu a. T) & (\mu\text{-abstraction}) \\
& | & a\,(T) & \text{(naming)} \\
& | & \langle T \rangle & \text{(reset)}
\end{array}
$$

The set of values is defined as follows:

$$V \ ::= \ c\,V \ldots V \ | \ x \ | \ \lambda x. T$$

Intended operational meaning

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$$(\mu a. \ldots a\,(t_1) \ldots a\,(t_2) \ldots a\,(t_n) \ldots) \quad u$$

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$$\underbrace{(\mu a.\ \ldots a\,(t_1)\ldots a\,(t_2)\ldots a\,(t_n)\ldots)}_{\alpha\to\beta}\quad u$$

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$$\underbrace{(\mu a.\ \ldots a\,(t_1)\ldots a\,(t_2)\ldots a\,(t_n)\ldots)}_{\alpha\to\beta}\ \underbrace{u}_{\alpha}$$

Intended operational meaning

$$\underbrace{(\mu a.\ \ldots a\,\overbrace{(t_1)}^{\alpha\to\beta}\ldots a\,\overbrace{(t_2)}^{\alpha\to\beta}\ldots a\,\overbrace{(t_n)}^{\alpha\to\beta}\ldots)}_{\alpha\to\beta}\ \underbrace{u}_{\alpha}$$

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$$\underbrace{(\mu a.\ \ldots a\,\overbrace{(t_1)}^{\alpha\to\beta}\ldots a\,\overbrace{(t_2)}^{\alpha\to\beta}\ldots a\,\overbrace{(t_n)}^{\alpha\to\beta}\ldots)}_{\alpha\to\beta}\ \underbrace{u}_{\alpha}$$

$$\to\quad (\mu a.\ \ldots a\,(t_1\,u)\ldots a\,(t_2\,u)\ldots a\,(t_n\,u)\ldots)$$

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$$\to \quad (\mu a.\ \ldots a\,(t_1\,u) \ldots a\,(t_2\,u) \ldots a\,(t_n\,u) \ldots)$$

Symmetrically:

$$f\,(\mu a.\ \ldots a\,(t_1) \ldots a\,(t_2) \ldots a\,(t_n) \ldots) \quad \to \quad (\mu a.\ \ldots a\,(f\,t_1) \ldots a\,(f\,t_2) \ldots a\,(f\,t_n) \ldots)$$

Typing rules

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$$\Gamma, x : \alpha; \Delta^{\perp} \vdash x : \alpha$$

$$\frac{\Gamma, x : \alpha; \Delta^{\perp} \vdash t : \beta}{\Gamma; \Delta^{\perp} \vdash \lambda x.\, t : \alpha \to \beta} \qquad \frac{\Gamma; \Delta^{\perp} \vdash t : \alpha \to \beta \qquad \Gamma; \Delta^{\perp} \vdash u : \alpha}{\Gamma; \Delta^{\perp} \vdash t\, u : \beta}$$

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$$\frac{\Gamma;\, \Delta^{\perp},\, a : \alpha^{\perp} \vdash t : \perp}{\Gamma;\, \Delta^{\perp} \vdash \mu a.\, t : \alpha} \qquad \frac{\Gamma;\, \Delta^{\perp},\, a : \alpha^{\perp} \vdash t : \alpha}{\Gamma;\, \Delta^{\perp},\, a : \alpha^{\perp} \vdash a\, (t) : \perp}$$

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$$\frac{\Gamma; \Delta^{\perp} \vdash t : \perp}{\Gamma; \Delta^{\perp} \vdash \langle t \rangle : \perp}$$

Reduction rules

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$(\lambda x.\, t)\, v \rightarrow t[x{:=}v]$ (v is a value)

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Renaming:

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$\mu a.\, a\,(t) \rightarrow t$ (a does not occur in t)

Reduction rules

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$(\mu a.\, t)\, u \to \mu b.\, t[a\,(\_){:=}b\,(\_\, u)]$

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Representig Contexts as $\lambda\mu$-Terms

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Dynamic interpretation of atomic propositions

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Dynamic interpretation of atomic propositions

$$\overline{R\, t_1 \, \ldots \, t_n} = \lambda e.\, \mu c.\, R\, t_1 \, \ldots \, t_n \wedge c\,(e)$$

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Dynamic interpretation of atomic propositions

$$\overline{R\,t_1\,\ldots\,t_n} = \lambda e.\,\mu c.\,R\,t_1\,\ldots\,t_n \wedge c\,(e)$$

Indeed:

$$\dfrac{\dfrac{\dfrac{e:\gamma;\,c:\gamma^o \vdash R\,t_1\,\ldots\,t_n : o \qquad \dfrac{e:\gamma;\,c:\gamma^o \vdash e:\gamma}{e:\gamma;\,c:\gamma^o \vdash c\,(e):o}}{e:\gamma;\,c:\gamma^o \vdash R\,t_1\,\ldots\,t_n \wedge c\,(e):o}}{e:\gamma;\, \vdash \mu c.\,R\,t_1\,\ldots\,t_n \wedge c\,(e):\gamma}}{\vdash \lambda e.\,\mu c.\,R\,t_1\,\ldots\,t_n \wedge c\,(e):\gamma \to \gamma}$$

Composition of two dynamic propositions

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$$\lambda e.\, B\,(A\,e)$$

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$\lambda e.\, \overline{p_2}\,(\overline{p_1}\,e)$

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$$\lambda e.\, \overline{p_2}\,(\overline{p_1}\,e) \quad = \quad \lambda e.\,(\lambda e_2.\, \mu c_2.\, p_2 \wedge c_2\,(e_2))\,(\overline{p_2}\,e)$$

Composition of two dynamic propositions

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Example:

$$\begin{aligned}\lambda e.\, \overline{p_2}\,(\overline{p_1}\,e) &= \lambda e.\,(\lambda e_2.\,\mu c_2.\,p_2 \wedge c_2\,(e_2))\,(\overline{p_2}\,e)\\ &= \lambda e.\,(\lambda e_2.\,\mu c_2.\,p_2 \wedge c_2\,(e_2))\,((\lambda e_1.\,\mu c_1.\,p_1 \wedge c_1\,(e_1))\,e)\end{aligned}$$

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& = & \lambda e.\, (\lambda e_2.\, \mu c_2.\, p_2 \wedge c_2\,(e_2))\,((\lambda e_1.\, \mu c_1.\, p_1 \wedge c_1\,(e_1))\,e) \\
& \longrightarrow & \lambda e.\, (\lambda e_2.\, \mu c_2.\, p_2 \wedge c_2\,(e_2))\,(\mu c_1.\, p_1 \wedge c_1\,(e))
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& \longrightarrow & \lambda e.\, (\lambda e_2.\, \mu c_2.\, p_2 \wedge c_2\,(e_2))\,(\mu c_1.\, p_1 \wedge c_1\,(e)) \\
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&\longrightarrow \lambda e.\,(\lambda e_2.\,\mu c_2.\,p_2 \wedge c_2\,(e_2))\,(\mu c_1.\,p_1 \wedge c_1\,(e))\\
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& \longrightarrow & \lambda e.\,\mu c_1.\,p_1 \wedge (p_2 \wedge c_1\,(e))
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Reading a dynamic proposition

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$\mathsf{READ}\, e\, \overline{p}$

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& \longrightarrow & p \wedge \top \\
& \equiv & p
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Dynamic Logic Revisited

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Conjunction amounts to functional composition.

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Existential quantification introduces "reference markers" by updating the context:

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Negation

The scope of the negation must be restricted to the current proposition and, consequently, "reset" the continuation:

$$\sim A \triangleq \lambda e.\, \mu c.\, \neg(\mathsf{READ}\,e\,A) \wedge c\,(e)$$

Implication and Universal Quantification

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$$A \sqsupset B \triangleq \lambda e.\, \mu c.\, \neg\langle(\lambda e.\, \neg(\mathsf{READ}\, e\, B))\, (A\, e)\rangle \wedge c\,(e)$$

$$\Pi x.\, P\, x \triangleq \lambda e.\, \mu c.\, (\forall x.\, \mathsf{READ}\,(x{::}e)\,(P\, x)) \wedge c\,(e)$$

Donkey Sentence Again

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The dynamic lexical semantics is kept unchanged:

$$
\begin{aligned}
[\![\text{farmer}]\!] &= \overline{\textbf{farmer}} \\
[\![\text{donkey}]\!] &= \overline{\textbf{donkey}} \\
[\![\text{owns}]\!] &= \lambda OS.\, S\,(\lambda x.\, O\,(\lambda y.\, \overline{\textbf{own}}\, x\, y)) \\
[\![\text{beats}]\!] &= \lambda OS.\, S\,(\lambda x.\, O\,(\lambda y.\, \overline{\textbf{beat}}\, x\, y)) \\
[\![\text{who}]\!] &= \lambda RQx.\, Q\, x \sqcap R\,(\lambda P.\, P\, x) \\
[\![\text{a}]\!] &= \lambda PQ.\, \Sigma x.\, P\, x \sqcap Q\, x \\
[\![\text{every}]\!] &= \lambda PQ.\, \Pi x.\, P\, x \sqsupset Q\, x \\
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Then, we have that:

$$
[\![\text{beats}]\!]\; [\![\text{it}]\!]\; ([\![\text{every}]\!]\; ([\![\text{who}]\!]\; ([\![\text{owns}]\!]\; ([\![\text{a}]\!]\; [\![\text{donkey}]\!]))\; [\![\text{farmer}]\!]))
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$$[\![\text{beats}]\!]\,[\![\text{it}]\!]\,([\![\text{every}]\!]\,([\![\text{who}]\!]\,([\![\text{owns}]\!]\,([\![\text{a}]\!]\,[\![\text{donkey}]\!]))\,[\![\text{farmer}]\!]))$$

reduces to the following term :

$$\lambda e.\, \mu c.\, (\forall x.\, \mathbf{farmer}\, x \supset (\forall y.\, \mathbf{donkey}\, y \supset (\mathbf{own}\, x\, y \supset \mathbf{beat}\, x\, (\mathsf{sel}\,(x{::}y{::}e))))) \wedge c\,(e)$$

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- To this end, the use of the "reset" operator is central.
- The reset operator we have used is rather "cheap".

Conclusions

- The $\lambda\mu$-calculus allows propositions (o) and contexts (γ) to be mixed in a same term.
- To this end, the use of the "reset" operator is central.
- The reset operator we have used is rather "cheap".
- More powerful versions (w.r.t. typing) should allow other dynamic phenomena (e.g. definite clauses, focus, presuppositions) to be handled similarly.